

ON THE PROPERTY AND CONFIGURATION OF THE SHORT TIME DFT FEED-FORWARD SYLLABIC COMPANDOR

Short Time DFTフィードフォワード シラビックコンパンダの特性と構成

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ABSTRACT Reducing the spectrum occupancy over radio channels is as well known as important to prevent from exhausting radio spectrum resources. The syllabic compandor provides with indispensable function both to improve speech quality and reduce fading noise over radio channels against to saving transmission power and narrowing spectrum occupancy.

Therefore, many investigations are keenly studied on realizing the syllabic compandors. Unfortunately, most of them have been concerned with approximation in the way of AM demodulation to estimate envelope component with certain intermodulation error between input signals and their envelopes. When such approximating envelope detector is adopted to compressor/expander, the intermodulation error in the approximately estimated envelopes induces harmonic distortions to expand spectrum occupancy and to degrade speech quality in the aprior communication systems.

An exact compandor is successfully discussed with employing instantaneous spectrum analysis insted of AM demodulation approximations in realizing the envelope detectors, which is based on that compressing/expanding is precisely performed on the frequency domain.

After being decomposed into instantaneous spectrum, input signals are processed via dividing/multiplying along each vector of the instantaneous spectrum components. The output signals are synthesized from the compressed/expanded instantaneous spectrum without almost any harmonic distortions.

1. INTRODUCTION

As the syllabic compandor provides with indispensable function both to improve speech quality and reduce fading noise over radio channels against to saving transmission power and narrowing spectrum occupancy, many investigations are keenly studied on realizing

the syllabic compandors[1]. Unfortunately, most of them have been concerned with approximation in the way of AM demodulation to estimate envelope of input signals. When such approximated envelope detector is adopted to compressor/expander, the intermodulation error owed to approximate estimation induces harmonic distortions to expand spectrum occupancy and to degrade speech quality in the aprior

communication systems.

Instead of employing approximate envelope detector of AM demodulators, the previously reported short time DFT is adopted to analyze input signals into the instantaneous spectrum [2, 3]. The time variant real and imaginary parts of each instantaneous spectrum component are interpreted as a kind of time variant coefficient of the Fourier series with exception of possessing common time variant phase function. Where input signals are decomposed into instantaneous spectrum, the envelope is equal to the absolute of the vector spanned by both real and imaginary parts of each instantaneous spectrum component. Dividing/multiplying input signals in prior to their envelope to get compressing/expanding 2 to 1/1 to 2 in dB meanings becomes to the individual processing of dividing/multiplying the spectrum components along to the spanned complex vector on the frequency domain, respectively. That is, compressing/expanding individually the instantaneous spectrum of input signals along to the spectrum component vector is the only way to avoid any phase deviation in compressing/multiplying input signals.

This new compandor proposed in this paper, named by "short time DFT feed-forward syllabic compandor (ab. in ST-DFT compandor)", is precisely substantiated by both envelope characteristics being 1:2 in dB meaning and intermodulation error being eliminated to vanish almost all harmonis distortion through computer simulations.

A circuitry configuration of the ST-DFT compandor is categorized into three major blocks: I instantanuous spectrum analyzer, II frequency domain compressor/expander, III output signal synthesizer. It is easy to understand that the ST-DFT compandor is ideal and free from any distortion, while the instantanuous spectrum analysis is performed without any errors.

2. INSTANTANEOUS SPECTRUM AND SHORT TIME DFT

The most significant concept in the ST-DFT compandor is the instantaneous spectrum. Now, we consider how the short time DFT gives the instantaneous spectrum.

Let the instantaneous spectrum $\Phi(n)$ at sampling time n be given by,

$$\Phi(n) = \{\phi_0(n) \ \phi_1(n) \ \phi_2(n) \ \cdots \ \phi_{N-1}(n)\}^T. \quad (1)$$

Where, $\phi_k(n)$ is a spectrum component at frequency index k of $\Phi(n)$.

The spectrum component $\phi_k(n)$ is defined by the short time DFT as follows,

$$\phi_k(n) = \sum_{r=-\infty}^{\infty} x(r) h(n-r) W_N^{-rk}, \quad (2)$$

here, $x(r)$ is an input data at sampling time r , W_N^{-rk} is the same operator defined in the existing DFT as follows,

$$W_N^{-rk} = \exp(-j \frac{2\pi rk}{N}),$$

integer k is $0 \leq k < N$.

Existence of ST-DFT requires that output signal $y(n)$ at time n is precisely reproduced from the instantanuous spectrum $\Phi(n)$ via ST-IFT. That is,

$$y(n) = \frac{1}{N} \sum_{k=0}^{N-1} \phi_k(n) W_N^{nk},$$

$$W_N^{nk} = \exp(j \frac{2\pi nk}{N}). \quad (3)$$

Here, W_N^{nk} is given by the same operator as existing inverse DFT.

The ST-IFT denoted by eq.3 requires that the window function $h(*)$ of eq.2 holds the condition, $y(n)=x(n)$. Substituting eq.2 into eq.3 and exchanging the summation order for variables k and r , it gives eq.4.

$$\begin{aligned} y(n) &= \frac{1}{N} \sum_{k=0}^{N-1} \{ \sum_{r=-\infty}^{\infty} x(r) h(n-r) W_N^{-rk} \} W_N^{nk} \\ &= \sum_{r=-\infty}^{\infty} x(r) h(n-r) \{ \frac{1}{N} \sum_{k=0}^{N-1} W_N^{(n-r)k} \}. \quad (4) \end{aligned}$$

The summation for variable k takes non zero value by N , only if $n-r = 2Nq$. Here, q is integer. This gives the following restrict condition to the window function.

$$h(p) = \begin{cases} 1, & \text{if } p = 0 \\ 0, & \text{if } p = 2Nu, \end{cases} \quad (5)$$

u is non zero integer.

For example, an N frame length Nyquist window function truncated with $2m$ frame number $h(p)$,

$$h(p) = \frac{\sin(p\pi/N)}{p\pi/N}, \quad -mN \leq p \leq mN, \quad (6)$$

are able to satisfy eq.5. Hereafter, the truncated Nyquist will be employed for the present as a window function in the ST-DFT.

3. ST-DFT COMPANDOR AND ITS PRINCIPLE

The processing outline in the ST-DFT compandor is shown in fig.1. In the ST-DFT compandor, input signal $x(r)$ is at first analyzed by ST-DFT

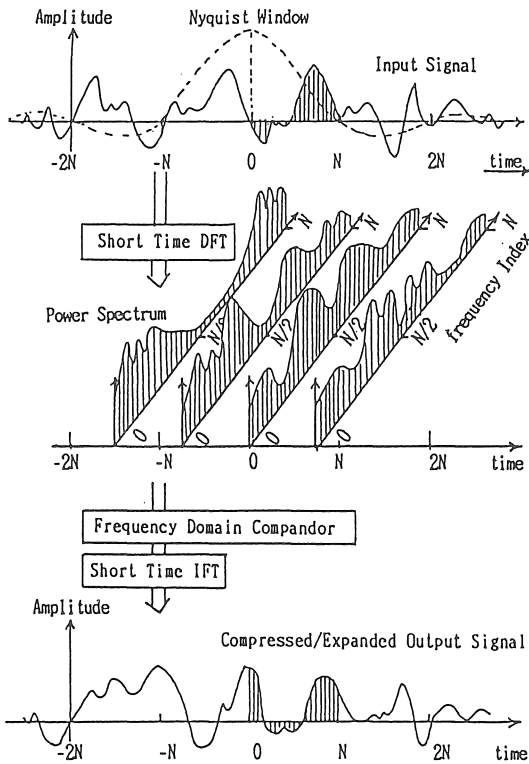


Fig.1 Processing Outline in the ST-DFT Compandor

to yield instantaneous spectrum $\Phi(n)$. Secondly, both real and imaginary parts of each component of the instantaneous spectrum are dividing/multiplying by prior amount to get compressed/expanded spectrum $\tilde{\Phi}(n)$. Output signal $\tilde{y}(n)$ is finally synthesized through the ST-IFT from compressed/expanded spectrum $\tilde{\Phi}(n)$.

So long as being concerned with the instantaneous spectrum, output signals are seemed to be given by a kind of the Fourier series with time variant complex coefficients as shown in eq.3, where $\phi_k(n)$ mean time variant complex coefficients and W_N^{nk} mean harmonics carriers of the Fourier series. The envelope of arbitrary signals is precisely defined by the magnitude of the instantaneous spectrum component $\phi_k(n)$. The companded instantaneous spectrum component $\tilde{\phi}_k(n)$ are given as follows.

$$\tilde{\phi}_k(n) = |\phi_k(n)| \alpha e^{j\theta},$$

$$\theta(n) = \tan^{-1} [b_k(n)/a_k(n)]. \quad (7)$$

Here, $\alpha=0.5$ for compressing, $\alpha=2$ for expanding, $a_k(n)$ is real part of $\phi_k(n)$, and $b_k(n)$ is imaginary of that.

The real part $\tilde{a}_k(n)$ or imaginary part $\tilde{b}_k(n)$ of compressed/expanded instantaneous spectrum component $\tilde{\phi}_k(n)$ is given by eq.8.

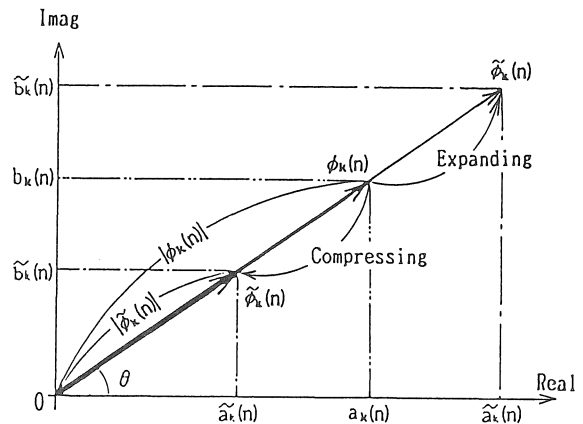


Fig.2 Schema Concept of the compressing/expanding on the frequency domain.

$$\begin{aligned}\tilde{a}_k(n) &= \left| \phi_k(n) \right|^{\alpha-1} a_k(n), \\ \tilde{b}_k(n) &= \left| \phi_k(n) \right|^{\alpha-1} b_k(n),\end{aligned}\quad (8)$$

$$\begin{aligned}\text{Here, } \cos \theta(n) &= a_k(n) / \left| \phi_k(n) \right|, \\ \sin \theta(n) &= b_k(n) / \left| \phi_k(n) \right|.\end{aligned}$$

Proof: *Preciseness of Companding on the Frequency Domain*

On the assumption that the companded instantaneous spectrum $\tilde{\Phi}(n)$ yields pure real signals, the frequency domain compander is easily understood to be perfect in companding signals and to be free from any distortions. Convergence of the output signals being pure real is soon shown in the follows.

Equation 7 gives compressing operation, if α is 0.5, and expanding, if α is 2. Where α is 1, the output signals are precisely equal to input signals. Substituting $\tilde{\phi}_k(n)$ with $\phi_k(n)$ of eq.3, output signals $\tilde{y}(n)$ are given as follows.

$$\begin{aligned}\tilde{y}(n) &= \frac{1}{N} \sum_{k=0}^{N-1} \tilde{\phi}_k(n) W_N^{nk} \\ &= \frac{1}{N} \{ \tilde{\phi}_0(n) W_N^0 + \tilde{\phi}_{N/2}(n) W_N^{nN/2} \} \\ &\quad + \frac{1}{N} \sum_{k=1}^{N/2-1} \{ \tilde{\phi}_k(n) W_N^{nk} + \tilde{\phi}_{N-k}(n) W_N^{n(N-k)} \}\end{aligned}\quad (9)$$

In eq.9, W_N^0 or $W_N^{nN/2}$ takes pure real 1 or 1(-), respectively. If $\phi_0(n)$ or $\phi_{N/2}(n)$ is pure real, the corresponding companded component $\tilde{\phi}_0(n)$ or $\tilde{\phi}_{N/2}(n)$ takes pure real value. Since $W_N^{n(N-k)}$ is given by complex conjugate with W_N^{nk} , $W_N^{n(N-k)} = \overline{W_N^{nk}}$, every bracketted term of eq.9 takes a pure real number, iff $\tilde{\phi}_{N-k}(n)$ is complex conjugate with $\tilde{\phi}_k(n)$. In practice, $\phi_k(n)$ holds the following relations,

$$\phi_0(n) = \sum_{r=-\infty}^{\infty} x(r)h(n-r) = \text{pure real}, \quad (10)$$

$$\phi_{N/2}(n) = \sum_{r=-\infty}^{\infty} x(r)h(n-r) (-1)^r = \text{pure real}, \quad (11)$$

$$\begin{aligned}\phi_{N-k}(n) &= \sum_{r=-\infty}^{\infty} x(r)h(n-r) \exp\{-j \frac{2\pi r(N-k)}{N}\} \\ &= \sum_{r=-\infty}^{\infty} x(r)h(n-r) \exp(j \frac{2\pi rk}{N}) = \overline{\phi_k(n)}.\end{aligned}\quad (12)$$

Both hands being multiplied by the same scalar, absolute of $\phi_k(n)$ or absolute of $\phi_{N-k}(n) = \overline{\phi_k(n)}$, eq.12 gives followings.

$$\begin{aligned}\tilde{\phi}_k(n) &= \left| \phi_k(n) \right|^{\alpha-1} \phi_k(n) \\ &= \left| \phi_{N-k}(n) \right|^{\alpha-1} \phi_{N-k}(n) = \overline{\tilde{\phi}_{N-k}(n)}\end{aligned}\quad (13)$$

Therefore, the compressed/expanded output signals $\tilde{y}(n)$ are finally given as pure real as follows.

$$\begin{aligned}\tilde{y}(n) &= \frac{1}{N} \sum_{k=0}^{N-1} \tilde{\phi}_k(n) W_N^{nk} \\ &= \frac{1}{N} \{ \tilde{\phi}_0(n) + (-1)^n \tilde{\phi}_{N/2}(n) \} \\ &\quad + \frac{1}{N} \sum_{k=1}^{N/2-1} \{ \tilde{\phi}_k(n) W_N^{nk} + \overline{\tilde{\phi}_k(n)} \overline{W_N^{nk}} \} \\ &= \frac{1}{N} \{ \tilde{\phi}_0(n) + (-1)^n \tilde{\phi}_{N/2}(n) \} \\ &\quad + \frac{2}{N} \sum_{k=1}^{N/2-1} 2\text{Real} \{ \tilde{\phi}_k(n) W_N^{nk} \}, \quad \text{QED.} \quad (14)\end{aligned}$$

4. CIRCUITRY CONFIGURATION

The ST-DFT compandor consists of three major blocks in processing wise, or $N/2+1$ sub-modules in frequency index wise, as shown in fig. 3, or as given by eq.14. The first block is the ST-DFT analyzer and consists of $N/2+1$ modules in which every component $\phi_k(n)$ is yielded. Inner product of $x(n)$ and W_N^{-rk} in eq.2 is performed of modulating the input $x(n)$ with complex carrier W_N^{-rk} of $2\pi k/N$ normalized angular frequency. Convolution $= \{x(r) W_N^{-rk}\}$ and $h(r)$ in eq.2 is also interpreted as low-pass filtering the modulated signal $\{x(r) W_N^{-rk}\}$ by $h(r)$. That is, the lower side band component of modulated signals $x(r) W_N^{-rk}$ is extracted to produce base band component $\phi_k(n)$ of instantaneous spectrum $\Phi(n)$.

The second block is a compressor/expander on the frequency domain as shown in detail in fig.4. This block is dominant in function, of which the compressor is implemented as shown in figs.4(a) (b), or the expander is implemented as shown in figs.4(c) (d), respectively.

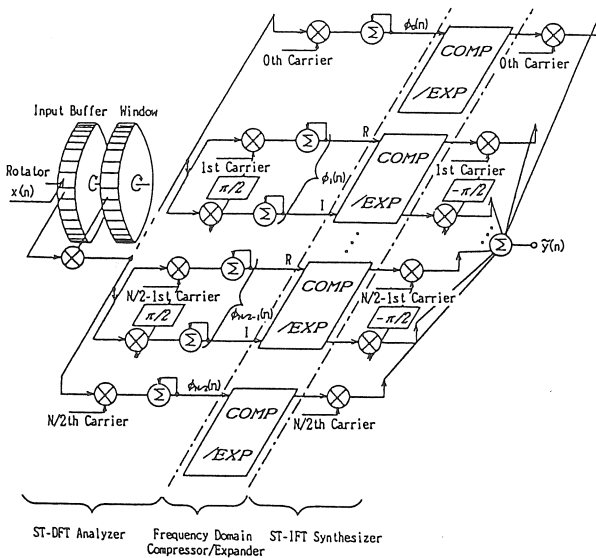


Fig.3 Configuration of the ST-DFT Compandor

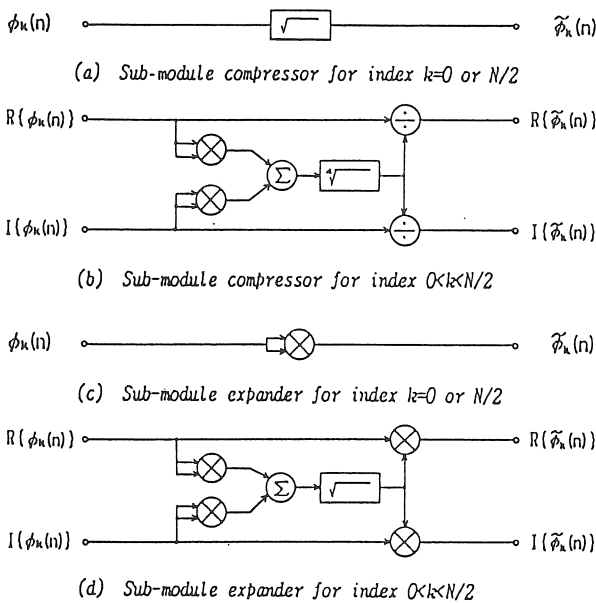


Fig.4 Detailed schemes of the frequency domain compressor(a), (b), and expander(c), (d).

According to the 0th or N/2th instantaneous spectrum component being pure real as given by eqs.10 or 11, 0th or N/2th sub-module of the frequency domain compressor/expander requires merely scalar functional element of a router/multiplier as shown in figs.4(a) or 4(c).

The part with common topology in both figs.4(b) and 4(d), which consists of two multipliers and an adder, means function of calculating square amplitude of the instantaneous spectrum component $\phi_k(n)$. In fig. 4(b), the part which consists of $\sqrt{\cdot}$ and dividers compresses the instantaneous spectrum $\phi_k(n)$ on the frequency domain in proportion to the root absolute $\phi_k(n)$. In fig.4(d), the part consists of a router and multipliers expands the instantaneous spectrum component $\phi_k(n)$ in proportion to square absolute $\phi_k(n)$. The sub-modules for frequency index $N/2 < k < N$ are eliminated from the implementation of ST-DFT compander with regard of $\phi_{N-k}(n)$ being complex conjugate with $\phi_k(n)$.

The last is a ST-IFT synthesizer to produce time domain compressed/expanded signals. In similar to the first block, ST-IFT synthesizer plays the roll of modulating compressed/expanded spectrum component $\phi_k(n)$ with complex carrier W_N^{nk} .

5. EXPERIMENT RESULTS

Operating Characteristics

The ST-DFT compandor is substantiated to be ideal in companding input signals through computer simulations on CRAY X-MP14se at AIT to separate from any round-off errors. Basically, the ST-DFT compandor excludes any IIR filters and feedback-loops, amplitude of companded signals through the ST-DFT compandors is precisely compressed/expanded in 2 to 1/1 to 2 in dB meanings with almost equivalent level to the granulating error for tonal signals.

Noise Reduction Effect

Figure 5 indicates noise reduction effect via the ST-DFT companders, where $2m=8$ and $N=32$ both in the ST-DFT compressor and expander. As well known, syllabic compander operate to improve speech quality transmitted over poor channels with installing both compressor at transmission front-end and expander at receiving tail-end. The original voice signals are observed as shown in fig.5(a) to be compressed via ST-DFT compressor. Fig.5(b) shows transmitted signals with -10 dB rms noise observed at the output terminal of the demodulator. Dominant noise shown in fig.5(b) is thermal white noise interfered over poor channels and transmission equipment. Expanded signals shown in fig.5(c) are sufficiently recovered via the ST-DFT expander from the interfered noise.

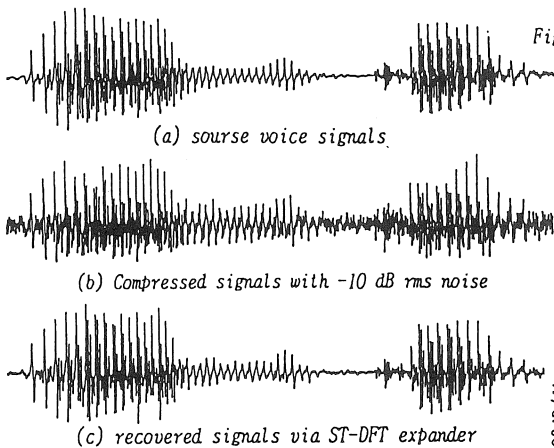


Fig.5 Noise reduction effect via the ST-DFT compressor and expander.

Transient Response

Transient response is examined under CCITT REC.G162 specification as shown in fig.6, here $2m=8$, $N=32$ in the ST-DFT companders. Source 2kHz tone-burst signals with 12 dB step are pictured in fig.6(a), whose greater amplitude is set to be -4 dBm and smaller is set to be -16 dBm. The attack time t_a is defined by the settling time after sudden change being applied

to within 150 % of steady-state value, the recover time t_r is defined by the settling time after sudden change being applied to within 75 % of steady-state value, t_a and t_r should be within 5 ms and 22.5 ms. Fig.6(b) shows that both t_a and t_r vanish to zero in ST-DFT compressor response. Expanding the compressed signals gives the ideal transient response as shown in fig.6(c), where t_a and t_r are disable from observation.

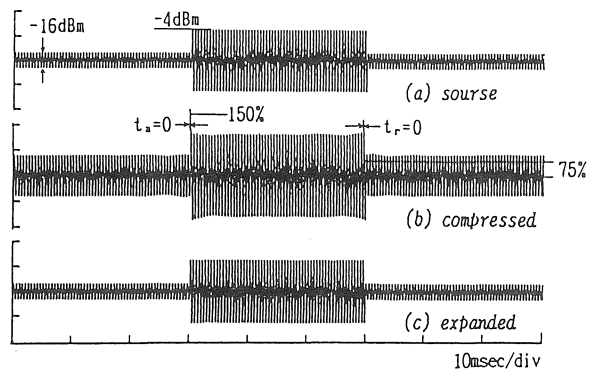


Fig.6 Transient Response for the 2kHz tone-burst with 12dB step, (a)original source, (b)output of the ST-DFT compressor, (c)output of the combined ST-DFT compressor-expander.

Harmonic Distortion

Harmonic distortion, measured with 800 Hz 0 dBm tonal signals, is recommended to be below 4 % (-14 dB). Fig.7(a) shows clearly that the

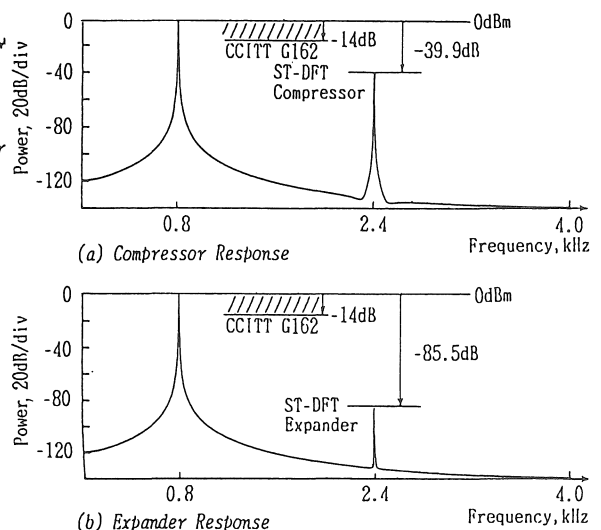


Fig.7 Frequency response of (a)ST-DFT compressor and (b)expander for 800Hz 0dBm tonal signals

maximum distortion in the power spectrum appears at 2.4 kHz as the third harmonic below -39.9 dBm, and the second at 4.0 kHz as the fifth below -135dBm. The harmonic distortion of the ST-DFT compressor is observed to be below -39 dB with more than 25 dB margin to the -14 dB limit. Fig.7(b) shows the frequency response of ST-DFT expander which operate separately. In fig.7(b), the maximum distortion appears at 2.4 kHz as the third harmonic below -85.5 dBm, and the second at 4.0 kHz as the fifth below -138dBm. The harmonic distortion of the ST-DFT expander is observed to be below -85 dB with more than 70 dB margin to the -14 dB limit.

Intermodulation

The intermodulated signal level, which seems adequate for Signalling System No.5, is also recommended to be below -26 dB at frequency $2f_1-f_2 (=f_L)$ and $2f_2-f_1 (=f_U)$ for compressor and expander separately. Here, input signals f_1 and f_2 are 900 Hz and 1020 Hz at a level of -5 dBm or -15 dBm. It is shown in fig.8(a) for the input signals specified in aboves that the error intermodulated in the ST-DFT compressor

separetely is at a level of below -30 dB when $N=32$, that the intermodulation error is improved to be at a level of below -40 dB when $N=64$ more than 32, and that both intermodulation errors does not exceed -26 dB of the CCITT compressor limit at frequencies f_L and f_U . It is also shown in fig.8(b) for the same specified signals that the error intermoduled in the ST-DFT expander separetely is at a level of below -35 dB when $N=32$ and at -45 when $N=64$, and does not exceed -26 dB of CCITT compressor limit.

6. CONCLUSION

A noble compandor was discussed with emphasis on the instantaneous spectrum signal processing, through its circuitry configuration, noise reduction effect, transient response, harmonic distortion, and intemodulation. Being adopted a primitive truncated Nyquit for the significant window $h(*)$, ST-DFT compandor is able to be almost free from any distortion both in steady-state frequency response and transient response.

Farther studies on optimizing the window function will improve the accuracy of the instantaneous spectrum signal processing and ST-DFT compandors.

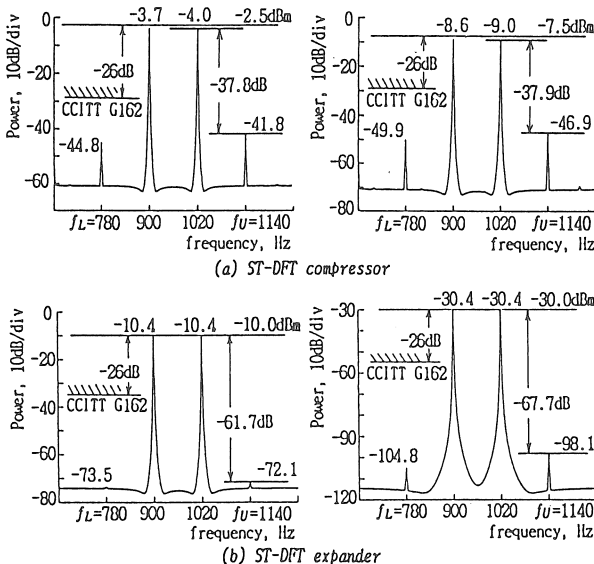


Fig.8 Frequency responses of the ST-DFT compressor(a), ST-DFT expander(b) separately, -5dBm for the left column, -15dBm for the right column, $N=64$ and quasi ideal proto-type filter

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