剛体床上に固定された弾性板に等分布荷重が作用する場合の応力解析

水野光国・中島夏樹

Stress Analysis of an Infinite Elastic Plate Fixed upon a Rigid Foundation and Subjected to a Uniformly Distributed Load.

Mitsukuni Mizuno and Natsuki Nakajima

本論文では応力及び変位成分値の数値計算法につき，一提案を行なう．すなわちエアリーの応力関数を用い，境界面上に作用する荷重をフーリェ展開し，無限積分形で表わされた応力関数の双曲線関数部分を指数関数で表わし，分母をマクローリン展開する．これより，各応力及び変位成分を求め，境界条件を近似的に満足するように連立方程式を解くことにより，実用上十分な精度の近似値を求め得ることを示す．

1. 緒　言

一端が剛体床上に固定され，他端に荷重を受ける弾性板の応力解析は，フーリェ変換を用いて解析可能である事は，知られており数値計算も行なわれている[1]．しかし各応力成分，変位成分式は，無限積分形を取るため，電子計算機を使用し，数値積分法を用いれば数値計算は可能であるが，負荷端近傍では収束が遅く，多くの時間を要する．先に集中荷重を受ける場合[2]で行ったと類似の方法[3)4)]を用い，等分布荷重を受ける場合を例にとり，短時間で，実用上十分な精度の近似解を求め得ることを示す．すなわち，等分布荷重が作用する場合を例にとり，各応力成分，変位成分を求め，項別積分を行い，初めの数項を取り，境界条件を近似的に満足させるように操作し，各応力成分値の数値計算を行なう．

2. 応力関数

x を横軸に，y を縦軸に取るとエアリーの応力関数は次式を満足せねばならない．

$$\frac{\partial^4 F}{\partial X^4}+2\frac{\partial^4 F}{\partial X^2 \partial y^2}+\frac{\partial^4 F}{\partial y^4}=0 \tag{1}$$

各応力成分は次のように表わされる．

$$\sigma_x=\frac{\partial^2 F}{\partial y^2},\ \sigma_y=\frac{\partial^2 F}{\partial X^2}\quad \tau=-\frac{\partial^2 F}{\partial X \partial y} \tag{2}$$

また，歪成分 ε_x, ε_y は ν をポアソン比，E をヤング率とすれば，

$$\left.\begin{aligned} E\varepsilon_x&=(1-\nu^2)\,\sigma_x-\nu\,(1+\nu)\,\sigma_y\\ E\varepsilon_y&=(1-\nu^2)\,\sigma_y-\nu\,(1+\nu)\,\sigma_x\end{aligned}\right\} \tag{3}$$

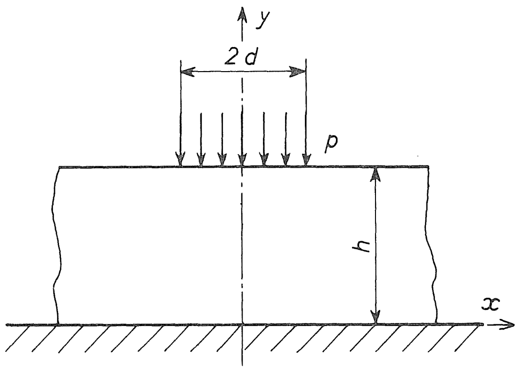

Fig. 1. Infinite elastic plate fixed upon a rigid foundation and subjected to uniformly distributed loads

ここで，Fig 1 の負荷状態即ち，高さ h の弾性板の一端が剛体床上に固定され，他端に y軸に対称な等分布荷重が作用している時式(1)を満足する解から次のように Fをとる．

$$F=\sum_{n=1}^{\infty}(A_n\cosh\alpha_n y+B_n\sinh\alpha_n y+C_n y\cosh\alpha_n y+D_n y\sinh\alpha_n y)\cos\alpha_n x \tag{4}$$

ここに，A_n, B_n, C_n, D_nは境界条件より定まる定数でありα_nは

$$\alpha_n=\frac{n\pi}{\ell},\quad n=1,2,3,\cdots\cdots \tag{5}$$

であり，ℓは展開の幅を表わす．境界面上に作用する垂直荷重$-p(x)$をフーリェ級数に展開する．y＝ hでは

$$\sigma_y = \sum_{n=1}^{\infty} 2\left\{f(p)\cos\alpha_n x\right\}/\ell \tag{6}$$

ただし

$$f(p) = \int_0^{\ell} p(x)\cos\alpha_n x dx \tag{7}$$

いま，x 軸方向の変位を u，y 軸方向の変位を v とすれば，

$$EU = \int E\varepsilon_x \partial x,\quad EV = \int E\varepsilon_y \partial y \tag{8}$$

ここで，積分により生ずる xのみの未知関数，y のみの未知関数は，境界条件より0となる．A_n，B_n，C_n，D_n を境界条件より定める．y=0 で変位は0，y=h では剪断力が0である．又荷重は等分布である．以上より次式を得る．

$$F = \sum_{n=1}^{\infty} \frac{2}{\ell\alpha_n^2}\Big[\left\{\alpha_n h \sinh\alpha_n h + 2(1-\nu)\cosh\alpha_n h\right\}\left\{2(1-\nu)\cos\alpha_n y - \alpha_n y \sinh\alpha_n y\right\}$$
$$+\left\{\alpha_n h\cosh\alpha_n h - (1-2\nu)\sinh\alpha_n h\right\}\left\{(1-2\nu)\sinh\alpha_n y + \alpha_n y\cosh\alpha_n y\right\}\Big]$$
$$\times\left\{4(1-\nu)^2\cosh^2\alpha_n h - (1-2\nu)^2\sinh^2\alpha_n h + \alpha_n^2 h^2\right\}^{-1} f(p)\cos\alpha_n x \tag{9}$$

式(5)より$\Delta n = \ell\Delta\alpha_n/\pi\ (=1)$ を式(9)の左右両辺に乗じ積分形に変える．x/h=X，y/h=Y，$\alpha_n h=\lambda$とおけば，

$$F = \frac{2h}{\pi}\int_0^{\infty}\frac{1}{\lambda^2}\Big[\left\{\lambda\sinh\lambda + 2(1-\nu)\cosh\lambda\right\}\left\{2(1-\nu)\cosh Y\lambda - Y\lambda\sinh Y\lambda\right\}$$
$$+\left\{\lambda\cosh\lambda - (1-2\nu)\sinh\lambda\right\}\left\{(1-2\nu)\sinh Y\lambda + Y\lambda\cosh Y\lambda\right\}\Big]$$
$$\times\left\{4(1-\nu)^2\cosh^2\lambda - (1-2\nu)^2\sinh^2\lambda + \lambda^2\right\}^{-1} f(p)\cos X\lambda\, d\lambda \tag{10}$$

3.　応力成分

Fig 1 の如く等分布荷重が作用する場合を考える．$-d \leqq x \leqq d$で$p(x)=p$，その他で$p(x)=0$，式(7)より，

$$f(p) = p\int_0^d \cos\alpha_n x\, dx = \frac{p}{\alpha_n}\sin\alpha_n d = \frac{ph}{\lambda}\sin\frac{d}{h}\lambda$$

全荷重を Pとおけば，P=2pd なるゆえ，

$$f(p) = \frac{Ph}{2d\lambda}\sin\frac{d}{h}\lambda \tag{11}$$

式(10)に代入すれば，

$$F = \frac{Ph}{\pi}\frac{h}{d}\int_0^{\infty}\frac{1}{\lambda^3}\Big[\left\{\lambda\sinh\lambda + 2(1-\nu)\cosh\lambda\right\}\left\{2(1-\nu)\cosh Y\lambda - Y\lambda\sinh Y\lambda\right\}$$
$$+\left\{\lambda\cosh\lambda - (1-2\nu)\sinh\lambda\right\}\left\{(1-2\nu)\sinh Y\lambda + Y\lambda\cosh Y\lambda\right\}\Big]$$
$$\times\left\{4(1-\nu)^2\cosh^2\lambda - (1-2\nu)^2\sinh^2\lambda + \lambda^2\right\}^{-1}\sinh\frac{d}{h}\lambda\cos X\lambda\, d\lambda$$

$2\sin\theta_1\cdot\cos\theta_2 = \sin(\theta_1-\theta_2) + \sin(\theta_1+\theta_2)$ より

$$F = \frac{Ph}{2\pi}\frac{h}{d}\int_0^{\infty}\frac{1}{\lambda^3}\Big[\left\{\lambda\sinh\lambda + 2(1-\nu)\cosh\lambda\right\}\left\{2(1-\nu)\cosh Y\lambda - Y\lambda\sinh Y\lambda\right\}$$
$$+\left\{\lambda\cosh\lambda - (1-2\nu)\sinh\lambda\right\}\left\{(1-2\nu)\sinh Y\lambda + Y\lambda\cosh Y\lambda\right\}\Big]$$
$$\times\left\{4(1-\nu)^2\cosh^2\lambda - (1-2\nu)^2\sinh^2\lambda + \lambda^2\right\}^{-1}\left\{\sin\left(\frac{d}{h}-X\right)\lambda + \sin\left(\frac{d}{h}+X\right)\lambda\right\}d\lambda \tag{12}$$

式(12)を式(2)に代入すれば，各応力成分を求め得るが，各式は無限積分形を取るため，数値計算には多くの時間を要する．そこで，数値計算法として，二，三の方法が提案されている．すなわち，被積分関数の分母を近似式で表わす方法，[1) 5)] 分母をマクローリン展開する方法等[3) 4)]である．本論文では，後者の方法を用いる．

3.1　被積分関数の分母のマクローリン展開

式(12)の分母をマクローリン展開する．

$$1/\left\{4(1-\nu)^2\cosh^2\lambda - (1-2\nu)^2\sinh^2\lambda + \lambda^2\right\}$$
$$=1/\left\{4(1-\nu)^2\sinh^2\lambda + 4(1-\nu)^2 - (1-2\nu)^2\sinh^2\lambda + \lambda^2\right\}$$
$$=1/\left\{(3-4\nu)\sinh^2\lambda + 4(1-\nu)^2 + \lambda^2\right\} = 2\left(X_1 e^{-2\lambda} + X_2 e^{-4\lambda} + X_3 e^{-6\lambda} + \cdots\right)/b$$

ただし

$$X_1=1,\quad X_2=-\frac{c+\lambda^2}{b}2,\quad X_3=\left(\frac{c+\lambda^2}{b}2\right)^2-1,\ \cdots\cdots;\ b=\frac{3-4\nu}{2},\quad c=\frac{5-12\nu+8\nu^2}{2}$$

分子を指数関数で表わせば,

$$\left[\left\{\lambda\sinh\lambda+2(1-\nu)\cosh\lambda\right\}\left\{2(1-\nu)\cosh Y\lambda-Y\lambda\sinh Y\lambda\right\}\right.$$
$$\left.+\left\{\lambda\cosh\lambda-(1-2\nu)\sinh\lambda\right\}\left\{(1-2\nu)\sinh Y\lambda+Y\lambda\cosh Y\lambda\right\}\right]$$
$$\times\left\{4(1-\nu)^2\cosh^2\lambda-(1-2\nu)^2\sinh^2\lambda+\lambda^2\right\}^{-1}$$

$$=\frac{2}{b}\left[\frac{b}{2}\left\{1+(1-Y)\lambda\right\}X_1e^{-(1-Y)\lambda}+\left(\frac{Y}{2}\lambda^2+\frac{1+Y}{4}\lambda+\frac{c}{2}\right)X_1e^{-(1+Y)\lambda}\right.$$
$$+\left[\left(\frac{Y}{2}\lambda^2-\frac{1+Y}{4}\lambda+\frac{c}{2}\right)X_1+\frac{b}{2}\left\{1+(1-Y)\lambda\right\}X_2\right]e^{-(3-Y)\lambda}$$
$$+\left[\frac{b}{2}\left\{1-(1-Y)\lambda\right\}X_1+\left(\frac{Y}{2}\lambda^2+\frac{1+Y}{4}\lambda+\frac{c}{2}\right)X_2\right]e^{-(3+Y)\lambda}$$
$$+\left[\left(\frac{Y}{2}\lambda^2-\frac{1+Y}{4}\lambda+\frac{c}{2}\right)X_2+\frac{b}{2}\left\{1+(1-Y)\lambda\right\}X_3\right]e^{-(5-Y)\lambda}$$
$$\left.+\left[\frac{b}{2}\left\{1-(1-Y)\lambda\right\}X_2+\left(\frac{Y}{2}\lambda^2+\frac{1+Y}{4}\lambda+\frac{c}{2}\right)X_3\right]e^{-(5+Y)\lambda}+\cdots\cdots\right]$$

いま, Φ を次のようにおく

$$\Phi_{\pm K}(x,y)=\int_0^\infty\lambda^{\pm k}e^{-y\lambda}\sin X\lambda d\lambda \tag{13}$$

ここで応力関数 F を表わす。

$$F=\frac{Ph}{\pi}\frac{1}{Db}(A_1+A_2+A_3+\ \cdots\cdots\cdots\ +A_6+\ \cdots\cdots\ ) \tag{14}$$

$$A_1=\frac{b}{2}\left[\left\{\Phi_{-3}(D-X,1-Y)+\Phi_{-3}(D+X,1-Y)\right\}+(1-Y)\left\{\Phi_{-2}(D-X,1-Y)\right.\right.$$
$$\left.\left.+\Phi_{-2}(D+X,1-Y)\right\}\right]$$

$$A_2=\frac{C}{2}\left\{\Phi_{-3}(D-X,1+Y)+\Phi_{-3}(D+X,1+Y)\right\}+\frac{1+Y}{4}\left\{\Phi_{-2}(D-X,1+Y)\right.$$
$$\left.+\Phi_{-2}(D+X,1+Y)\right\}+\frac{Y}{2}\left\{\Phi_{-1}(D-X,1+Y)+\Phi_{-1}(D+X,1+Y)\right\}$$

$$A_3=-C\left[\left\{\Phi_{-3}(D-X,3-Y)+\Phi_{-3}(D+X,3-Y)\right\}+(1-Y)\left\{\Phi_{-2}(D-X,3-Y)\right.\right.$$
$$\left.\left.+\Phi_{-2}(D+X,3-Y)\right\}\right]-\left[\left\{\Phi_{-1}(D-X,3-Y)+\Phi_{-1}(D+X,3-Y)\right\}\right.$$
$$\left.+(1-Y)\left\{\Phi_0(D-X,3-Y)+\Phi_0(D+X,3-Y)\right\}\right]+\frac{C}{2}\left\{\Phi_{-3}(D-X,3-Y)\right.$$
$$\left.+\Phi_{-3}(D+X,3-Y)\right\}-\frac{1+Y}{4}\left\{\Phi_{-2}(D-X,3-Y)+\Phi_{-2}(D+X,3-Y)\right\}$$
$$+\frac{Y}{2}\left\{\Phi_{-1}(D-X,3-Y)+\Phi_{-1}(D+X,3-Y)\right\}$$

$$A_4=\frac{b}{2}\left[\left\{\Phi_{-3}(D-X,3+Y)+\Phi_{-3}(D+X,3+Y)\right\}-(1-Y)\left\{\Phi_{-2}(D-X,3+Y)\right.\right.$$
$$\left.\left.+\Phi_{-2}(D+X,3+Y)\right\}\right]-\frac{2C}{b}\left[\frac{C}{2}\left\{\Phi_{-3}(D-X,3+Y)+\Phi_{-3}(D+X,3+Y)\right\}+\frac{1+Y}{4}\left\{\Phi_{-2}(D-X,3+Y)\right.\right.$$
$$\left.\left.+\Phi_{-2}(D+X,3+Y)\right\}+\frac{Y}{2}\left\{\Phi_{-1}(D-X,3+Y)+\Phi_{-1}(D+X,3+Y)\right\}\right]-\frac{2}{b}\left[\frac{C}{2}\left\{\Phi_1(D-X,3+Y)\right.\right.$$

$$+\Phi_{-1}(D+X,\ 3+Y)\Big\}+\frac{1+Y}{4}\Big\{\Phi_0(D-X,\ 3+Y)+\Phi_0(D+X,\ 3+Y)\Big\}+\frac{Y}{2}\Big\{\Phi_1(D-X,\ 3+Y)$$

$$+\Phi_1(D+X,\ 3+Y)\Big\}\Big]$$

$$A_5=\frac{b}{2}\Big(\frac{4C^2}{b^2}-1\Big)\Big[\Big\{\Phi_{-3}(D-X,\ 5-Y)+\Phi_{-3}(D+X,\ 5-Y)\Big\}+(1-Y)\Big\{\Phi_{-2}(D-X,\ 5-Y)$$

$$+\Phi_{-2}(D+X,\ 5-Y)\Big\}\Big]+\frac{4C}{b}\Big[\Big\{\Phi_{-1}(D-X,\ 5-Y)+\Phi_{-1}(D+X,\ 5-Y)\Big\}$$

$$+(1-Y)\Big\{\Phi_0(D-X,\ 5-Y)+\Phi_0(D+X,\ 5-Y)\Big\}\Big]+\frac{2}{b}\Big[\Big\{\Phi_1(D-X,\ 5-Y)$$

$$+\Phi_1(D+X,\ 5-Y)\Big\}+(1-Y)\Big\{\Phi_2(D-X,\ 5-Y)+\Phi_2(D+X,\ 5-Y)\Big\}\Big]$$

$$-\frac{2C}{b}\Big[\frac{C}{2}\Big\{\Phi_{-3}(D-X,\ 5-Y)+\Phi_{-3}(D+X,\ 5-Y)\Big\}-\frac{1+Y}{4}\Big\{\Phi_{-2}(D-X,\ 5-Y)$$

$$+\Phi_{-2}(D+X,\ 5-Y)\Big\}+\frac{Y}{2}\Big\{\Phi_{-1}(D-X,\ 5-Y)+\Phi_{-1}(D+X,\ 5-Y)\Big\}\Big]$$

$$-\frac{2}{b}\Big[\frac{C}{2}\Big\{\Phi_{-1}(D-X,\ 5-Y)+\Phi_{-1}(D+X,\ 5-Y)\Big\}-\frac{1+Y}{4}\Big\{\Phi_0(D-X,\ 5-Y)$$

$$+\Phi_0(D+X,\ 5-Y)\Big\}+\frac{Y}{2}\Big\{\Phi_1(D-X,\ 5-Y)+\Phi_{-1}(D+X,\ 5-Y)\Big\}\Big]$$

$$A_6=-C\Big[\Big\{\Phi_{-3}(D-X,\ 5+Y)+\Phi_{-3}(D+X,\ 5+Y)\Big\}-(1-Y)\Big\{\Phi_{-2}(D-X,\ 5+Y)$$

$$+\Phi_{-2}(D+X,\ 5+Y)\Big\}\Big]-\Big[\Big\{\Phi_{-1}(D-X,\ 5+Y)+\Phi_{-1}(D+X,\ 5+Y)\Big\}$$

$$-(1-Y)\Big\{\Phi_0(D-X,\ 5+Y)+\Phi_0(D+X,\ 5+Y)\Big\}\Big]+\Big(\frac{4C^2}{b^2}-1\Big)\Big[\frac{C}{2}\Big\{\Phi_{-3}(D-X,\ 5+Y)$$

$$+\Phi_{-3}(D+X,\ 5+Y)\Big\}+\frac{1+Y}{4}\Big\{\Phi_{-2}(D-X,\ 5+Y)+\Phi_{-2}(D+X,\ 5+Y)\Big\}$$

$$+\frac{Y}{2}\Big\{\Phi_{-1}(D-X,\ 5+Y)+\Phi_{-1}(D+X,\ 5+Y)\Big\}\Big]+\frac{8C}{b^2}\Big[\frac{C}{2}\Big\{\Phi_{-1}(D-X,\ 5+Y)$$

$$+\Phi_{-1}(D+X,\ 5+Y)+\frac{1+Y}{4}\Big\{\Phi_0(D-X,\ 5+Y)+\Phi_0(D+X,\ 5+Y)\Big\}$$

$$+\frac{Y}{2}\Big\{\Phi_1(D-X,\ 5+Y)+\Phi_1(D+X,\ 5+Y)\Big\}\Big]+\Big(\frac{2}{b}\Big)^2\Big[\frac{C}{2}\Big\{\Phi_1(D-X,\ 5+Y)$$

$$+\Phi_1(D+X,\ 5+Y)\Big\}+\frac{1+Y}{4}\Big\{\Phi_2(D-X,\ 5+Y)+\Phi_2(D+X,\ 5+Y)\Big\}$$

$$+\frac{Y}{2}\Big\{\Phi_3(D-X,\ 5+Y)+\Phi_3(D+X,\ 5+Y)\Big\}\Big]$$

....

3.2　無限積分形応力関数の極座標系への変換

式(13)で定義した無限積分は，つぎのように三角関数で表わすことができる．Fig 2 に示すような関係を用いれば

$$\int_0^\infty e^{-y\lambda}\sin x\lambda d\lambda=\frac{x}{x^2+y^2}=\frac{\sin\varphi}{r}$$

$$\int_0^\infty e^{-y\lambda}\cos x\lambda d\lambda=\frac{y}{x^2+y^2}=\frac{\cos\varphi}{r}$$

上式の左右両辺をx，yで偏微分，偏積分すれば，次の公式を得る．ここでxの奇関数をΦ，偶関数をΨで表わし，λ の指数をもってその接尾字とする．

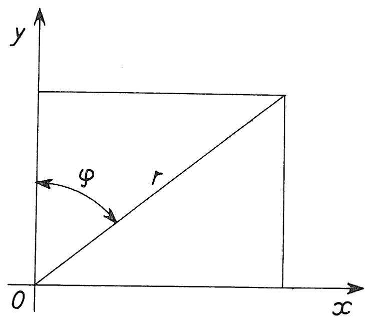

Fig. 2. Relation between rectangular and polar coordinates

$$
\left.\begin{aligned}
&\Phi_k(x,y)=\int_0^\infty \lambda^k e^{-y\lambda}\sin x\lambda\, d\lambda = k!\,\frac{\sin(k+1)\varphi}{r^{K+1}} && k=0,1,2,3,\cdots\\
&\Phi_{-k}(x,y)=\int_0^\infty \lambda^{-k} e^{-y\lambda}\sin x\lambda\, d\lambda=\frac{(-1)^{K-1}\, r^{K-1}}{(k-1)\,!}\Big\{\cos(k-1)\varphi\cdot\varphi\\
&+\sin(k-1)\varphi\log r-\Big(1+\frac{1}{2}+\cdots+\frac{1}{k-1}\Big)\sin(k-1)\varphi\Big\} && k=1,2,3,\cdots\\
&\Psi_k(x,y)=\int_0^\infty \lambda^k e^{-y\lambda}\cos x\lambda\, d\lambda = k!\,\frac{\cos(k+1)\varphi}{r^{K+1}} && k=0,1,2,3,\cdots\\
&\Psi_{-k}(x,y)=\int_0^\infty \lambda^{-k} e^{-y\lambda}\cos x\lambda\, d\lambda=\frac{(-1)^{k-1}\, r^{k-1}}{(k-1)\,!}\Big\{-\cos(k-1)\varphi\log r\\
&+\sin(k-1)\varphi\cdot\varphi+\Big(1+\frac{1}{2}+\cdots\cdots+\frac{1}{k-1}\Big)\cos(k-1)\varphi\Big\} && k=1,2,3,\cdots
\end{aligned}\right\} \quad (15)
$$

ただし$\left(1+\frac{1}{2}+\cdots+\frac{1}{k-1}\right)_{k=1}=0$ と定義する.

3.3 多極座標による応力成分

Fig 3のように記号を定め，また $K=P/\pi h$, $R=r/h$ とおくと，式(2) (14) (15) より，各応力成分は，つぎのように表わされる.

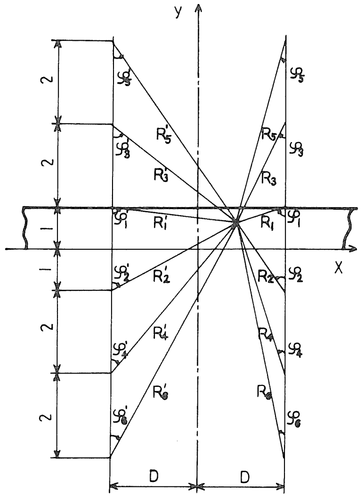

Fig. 3. Multi-polar coordinates

$$\sigma_x/K=(B_1+B_2+B_3+\cdots\cdots\cdots+B_6+\cdots\cdots)\,/\,Db \quad (16)$$

$$B_1=-\frac{b}{2}\Big\{(\varphi_1+\varphi'_1)-R_1\cos\varphi_1\Big(\frac{\sin\varphi_1}{R_1}+\frac{\sin\varphi'_1}{R'_1}\Big)\Big\}$$

$$B_2=\frac{C-2}{2}(\varphi_2+\varphi_2')-\frac{1}{4}R_3\cos\varphi_3\Big(\frac{\sin2\varphi_2}{R_2}+\frac{\sin\varphi'_2}{R'_2}\Big)+\frac{1}{2}(1-R_1\cos\varphi_1)\Big(\frac{\sin2\varphi_2}{R_2{}^2}+\frac{\sin2\varphi'_2}{R'_2{}^2}\Big)$$

$$B_3=C\Big\{(\varphi_3+\varphi'_3)-R_1\cos\varphi_1\Big(\frac{\sin\varphi_3}{R_3}+\frac{\sin\varphi'_3}{R_3}\Big)\Big\}+\Big(\frac{\text{si2n}\varphi_3}{R_3{}^2}+\frac{\sin2\varphi'_3}{R'_3{}^2}\Big)-2\,!\,R_1\cos\varphi_1\Big(\frac{\sin3\varphi_3}{R_3{}^3}+\frac{\sin3\varphi'_3}{R'_3{}^3}\Big)+\frac{C-1}{2}(\varphi_3+\varphi'_3)+\frac{1}{4}R_3\cos\varphi_3\Big(\frac{\sin\varphi_3}{R_3}+\frac{\sin\varphi'_3}{R'_3}\Big)+\frac{1}{2}(1-R_1\cos\varphi_1)\Big(\frac{\sin2\varphi_3}{R_3{}^2}+\frac{\sin2\varphi'_3}{R'_3{}^2}\Big)$$

$$B_4=-\frac{b}{2}\Big\{(\varphi_4+\varphi'_4)+R_1\cos\varphi_1\Big(\frac{\sin\varphi_4}{R_4}+\frac{\sin\varphi'_4}{R'_4}\Big)\Big\}-\frac{2C}{b}\Big\{\frac{C-1}{2}(\varphi_4+\varphi'_4)-\frac{1}{4}R_3\cos\varphi_3\Big(\frac{\sin\varphi_4}{R_4}+\frac{\sin\varphi'_4}{R'_4}\Big)+\frac{1}{2}(1-R_1\cos\varphi_1)\Big(\frac{\sin2\varphi_4}{R_4{}^2}+\frac{\sin2\varphi'_4}{R'_4{}^2}\Big)\Big\}-\frac{2}{b}\Big\{\frac{C-1}{2}\Big(\frac{\sin2\varphi_4}{R_4{}^2}+\frac{\sin2\varphi'_4}{R'_4{}^2}\Big)-\frac{2\,!}{4}R_3\cos\varphi_3\Big(\frac{\sin3\varphi_4}{R_4{}^3}+\frac{\sin3\varphi'_4}{R'_4{}^3}\Big)+\frac{3\,!}{2}(1-R_1\cos\varphi_1)\Big(\frac{\sin4\varphi_4}{R_4{}^4}+\frac{\sin4\varphi'_4}{R'_4{}^4}\Big)\Big\}$$

$$B_5=-\frac{b}{2}\Big(\frac{4C^2}{b^2}-1\Big)\Big\{(\varphi_5+\varphi'_5)-R_1\cos\varphi_1\Big(\frac{\sin\varphi_5}{R_5}+\frac{\sin\varphi'_5}{R'_5}\Big)\Big\}-\frac{4C}{b}\Big\{\Big(\frac{\sin2\varphi_5}{R_5{}^2}+\frac{\sin2\varphi'_5}{R'_5{}^2}\Big)-2\,!R_1\cos\varphi_1\Big(\frac{\sin3\varphi_5}{R_5{}^3}+\frac{\sin3\varphi'_5}{R'_5{}^3}\Big)\Big\}-\frac{2}{b}\Big\{+3!\Big(\frac{\sin4\varphi_5}{R_5{}^4}+\frac{\sin4\varphi'_5}{R'_5{}^4}\Big)-4\,!R_1\cos\varphi_1\Big(\frac{\sin5\varphi_5}{R_5{}^5}+\frac{\sin5\varphi'_5}{R'_5{}^5}\Big)\Big\}-\frac{2C}{b}\Big\{\frac{C-1}{2}(\varphi_5+\varphi'_5)+\frac{1}{4}R_3\cos\varphi_3\Big(\frac{\sin\varphi_5}{R_5}+\frac{\sin\varphi'_5}{R'_5}\Big)+\frac{1}{2}(1-R_1\cos\varphi_1)\Big(\frac{\sin2\varphi_5}{R_5{}^2}+\frac{\sin2\varphi'_5}{R'_5{}^2}\Big)\Big\}-\frac{2}{b}\Big\{\frac{C-1}{2}\Big(\frac{\sin2\varphi_5}{R_5{}^2}+\frac{\sin2\varphi'_5}{R'_5{}^2}\Big)+\frac{2\,!}{4}R_3\cos\varphi_3\Big(\frac{\sin3\varphi_5}{R_5{}^3}+\frac{\sin3\varphi'_5}{R'_5{}^3}\Big)+\frac{3\,!}{2}(1-R_1\cos\varphi_1)\Big(\frac{\sin4\varphi_5}{R_5{}^4}+\frac{\sin4\varphi'_5}{R'_5{}^4}\Big)\Big\}$$

$$B_6=C\Bigl\{(\varphi_6+\varphi'_6)+R_1\cos\varphi_1\Bigl(\frac{\sin\varphi_6}{R_6}+\frac{\sin\varphi'_6}{R'_6}\Bigr)\Bigr\}+\Bigl(\frac{\sin2\varphi_6}{R_6{}^2}+\frac{\sin2\varphi'_6}{R'_6{}^2}\Bigr)+2!R_1\cos\varphi_1\Bigl(\frac{\sin3\varphi_6}{R_6{}^3}+\frac{\sin3\varphi'_6}{R'_6{}^3}\Bigr)$$

$$+\Bigl(\frac{4c^2}{b^2}-1\Bigr)\Bigl\{\frac{C-1}{2}(\varphi_6+\varphi'_6)-\frac{1}{4}R_3\cos\varphi_3\Bigl(\frac{\sin\varphi_6}{R_6}+\frac{\sin\varphi'_6}{R'_6}+\frac{1}{2}(1-R_1\cos\varphi_1)\Bigl(\frac{\sin2\varphi_6}{R_6{}^2}+\frac{\sin2\varphi'_6}{R'_6{}^2}\Bigr)\Bigr\}$$

$$+\frac{8C}{b^2}\Bigl\{\frac{C-1}{2}\Bigl(\frac{\sin2\varphi_6}{R_6{}^2}+\frac{\sin2\varphi'_6}{R_6'^2}\Bigr)-\frac{2!}{4}R_3\cos\varphi_3\Bigl(\frac{\sin3\varphi_6}{R_6{}^3}+\frac{\sin3\varphi'_6}{R'_6{}^3}\Bigr)+\frac{3!}{2}(1-R_1\cos\varphi_1)\Bigl(\frac{\sin4\varphi_6}{R_6{}^4}+\frac{\sin4\varphi'_6}{R'_6{}^4}\Bigr)\Bigr\}$$

$$+\Bigl(\frac{2}{b}\Bigr)^2\Bigl\{3!\,\frac{C-1}{2}\Bigl(\frac{\sin4\varphi_6}{R_6{}^4}+\frac{\sin4\varphi'_6}{R'_6{}^4}\Bigr)-\frac{4!}{4}R_3\cos\varphi_3\Bigl(\frac{\sin5\varphi_6}{R_6{}^5}+\frac{\sin5\varphi'_6}{R'_6{}^5}\Bigr)$$

$$+\frac{5!}{2}(1-R_1\cos\varphi_1)\Bigl(\frac{\sin6\varphi_6}{R_6{}^6}+\frac{\sin6\varphi'_6}{R'_6{}^6}\Bigr)\Bigr\}$$

....

$$\sigma_y/K=-(C_1+C_2+C_3+\cdots\cdots+C_6+\cdots)/Db \qquad (17)$$

$$C_1=\frac{b}{2}\Bigl\{(\varphi_1+\varphi'_1)+R_1\cos\varphi_1\Bigl(\frac{\sin\varphi_1}{R_1}+\frac{\sin\varphi'_1}{R'_1}\Bigr)\Bigr\}$$

$$C_2=\frac{C}{2}(\varphi_2+\varphi'_2)+\frac{1}{4}R_2\cos\varphi_2\Bigl(\frac{\sin\varphi_2}{R_2}+\frac{\sin\varphi'_2}{R'_2}\Bigr)+\frac{1}{2}(1-R_1\cos\varphi_1)\Bigl(\frac{\sin2\varphi_2}{R_2{}^2}+\frac{\sin2\varphi'_2}{R'_2{}^2}\Bigr)$$

$$C_3=-C\Bigl\{(\varphi_3+\varphi'_3)+R_1\cos\varphi_1\Bigl(\frac{\sin\varphi_3}{R_3}+\frac{\sin\varphi'_3}{R'_3}\Bigr)\Bigr\}-\Bigl\{\Bigl(\frac{\sin2\varphi_3}{R_3{}^2}+\frac{\sin2\varphi'_3}{R'_3{}^2}\Bigr)+2!R_1\cos\varphi_1\Bigl(\frac{\sin3\varphi_3}{R_3{}^3}+\frac{\sin3\varphi'_3}{R'_3{}^3}\Bigr)\Bigr\}$$

$$+\frac{C}{2}(\varphi_3+\varphi'_3)-\frac{1}{4}R_2\cos\varphi_2\Bigl(\frac{\sin\varphi_3}{R_3}+\frac{\sin\varphi'_3}{R'_3}\Bigr)+\frac{1}{2}(1-R_1\cos\varphi_1)\Bigl(\frac{\sin2\varphi_3}{R_3{}^2}+\frac{\sin2\varphi'_3}{R'_3{}^2}\Bigr)$$

$$C_4=\frac{b}{2}\Bigl\{(\varphi_4+\varphi'_4)-R_1\cos\varphi_1\Bigl(\frac{\sin\varphi_4}{R_4}+\frac{\sin\varphi'_4}{R'_4}\Bigr)\Bigr\}-\frac{2C}{b}\Bigl\{\frac{C}{2}(\varphi_4+\varphi'_4)+\frac{1}{4}R_2\cos\varphi_2\Bigl(\frac{\sin\varphi_4}{R_4}+\frac{\sin\varphi'_4}{R'_4}\Bigr)$$

$$+\frac{1}{2}(1-R_1\cos\varphi_1)\Bigl(\frac{\sin2\varphi_4}{R_4{}^2}+\frac{\sin2\varphi'_4}{R'_4{}^2}\Bigr)\Bigr\}-\frac{2}{b}\Bigl\{\frac{C}{2}\Bigl(\frac{\sin2\varphi_4}{R_4{}^2}+\frac{\sin2\varphi'_4}{R'_4{}^2}\Bigr)+\frac{2!}{4}R_2\cos\varphi_2\Bigl(\frac{\sin3\varphi_4}{R_4{}^3}$$

$$+\frac{\sin3\varphi'_4}{R'_4{}^3}\Bigr)+\frac{3!}{2}(1-R_1\cos\varphi_1)\Bigl(\frac{\sin4\varphi_4}{R_4{}^4}+\frac{\sin4\varphi'_4}{R'_4{}^4}\Bigr)\Bigr\}$$

$$C_5=\frac{b}{2}\Bigl(\frac{4C^2}{b^2}-1\Bigr)\Bigl\{(\varphi_5+\varphi'_5)+R_1\cos\varphi_1\Bigl(\frac{\sin\varphi_5}{R_5}+\frac{\sin\varphi'_5}{R'_5}\Bigr)\Bigr\}+\frac{4C}{b}\Bigl\{\Bigl(\frac{\sin2\varphi_5}{R_5{}^2}+\frac{\sin2\varphi'_5}{R'_5{}^2}\Bigr)$$

$$+2!\,R_1\cos\varphi_1\Bigl(\frac{\sin3\varphi_5}{R_5{}^3}+\frac{\sin3\varphi'_5}{R'_5{}^3}\Bigr)\Bigr\}+\frac{2}{b}\Bigl\{3!\Bigl(\frac{\sin4\varphi_5}{R_5{}^4}+\frac{\sin4\varphi'_5}{R'_5{}^4}\Bigr)+4!R_1\cos\varphi_1\Bigl(\frac{\sin5\varphi_5}{R_5{}^5}+\frac{\sin5\varphi'_5}{R'_5{}^5}\Bigr)\Bigr\}$$

$$-\frac{2C}{b}\Bigl\{\frac{C}{2}(\varphi_5+\varphi'_5)-\frac{1}{4}R_2\cos\varphi_2\Bigl(\frac{\sin\varphi_5}{R_5}+\frac{\sin\varphi'_5}{R'_5}+\frac{1}{2}(1-R_1\cos\varphi_1)\Bigl(\frac{\sin2\varphi_5}{R_5{}^2}+\frac{\sin2\varphi'_5}{R'_5{}^2}\Bigr)\Bigr\}$$

$$-\frac{2}{b}\Bigl\{\frac{C}{2}\Bigl(\frac{\sin2\varphi_5}{R_5{}^2}+\frac{\sin2\varphi'_5}{R'_5{}^2}\Bigr)-\frac{2!}{4}R_2\cos\varphi_2\Bigl(\frac{\sin3\varphi_5}{R_5{}^3}+\frac{\sin3\varphi'_5}{R'_5{}^3}\Bigr)+\frac{3!}{2}(1-R_1\cos\varphi_1)$$

$$\times\Bigl(\frac{\sin4\varphi_5}{R_5{}^4}+\frac{\sin4\varphi'_5}{R'_5{}^4}\Bigr)\Bigr\}$$

$$C_6=-C\Bigl\{(\varphi_6+\varphi'_6)-R_1\cos\varphi_1\Bigl(\frac{\sin\varphi_6}{R_6}+\frac{\sin\varphi'_6}{R'_6}\Bigr)\Bigr\}-\Bigl(\frac{\mathrm{siu}2\varphi_6}{R_6{}^2}+\frac{\sin2\varphi'_6}{R'_6{}^2}\Bigr)$$

$$+2!R_1\cos\varphi_1\Bigl(\frac{\sin3\varphi_6}{R_6{}^3}+\frac{\sin3\varphi'_6}{R'_6{}^3}\Bigr)+\Bigl(\frac{4C^2}{b^2}-1\Bigr)\Bigl\{\frac{C}{2}(\varphi_6+\varphi'_6)+\frac{1}{4}R_2\cos\varphi_2\Bigl(\frac{\sin\varphi_6}{R_6}+\frac{\sin\varphi'_6}{R'_6}\Bigr)$$

$$+\frac{1}{2}(1-R_1\cos\varphi_1)\Bigl(\frac{\sin2\varphi_6}{R_6{}^2}+\frac{\sin2\varphi'_6}{R'_6{}^2}\Bigr)\Bigr\}+\frac{8C}{b^2}\Bigl\{\frac{C}{2}\Bigl(\frac{\sin2\varphi_6}{R_6{}^2}+\frac{\sin2\varphi'_6}{R'_6{}^2}\Bigr)+\frac{2!}{4}R_2\cos\varphi_2\Bigl(\frac{\sin3\varphi_6}{R_6{}^3}+\frac{\sin3\varphi'_6}{R'_6{}^3}\Bigr)$$

$$+\frac{3!}{2}(1-R_1\cos\varphi_1)\Bigl(\frac{\sin4\varphi_6}{R_6{}^4}+\frac{\sin4\varphi'_6}{R'_6{}^4}\Bigr)\Bigr\}+\Bigl(\frac{2}{b}\Bigr)^2\Bigl\{3!\,\frac{C}{2}\Bigl(\frac{\sin4\varphi_6}{R_6{}^4}+\frac{\sin4\varphi'_6}{R'_6{}^4}\Bigr)$$

$$+\frac{4!}{4}R_2\cos\varphi_2\Bigl(\frac{\sin5\varphi_6}{R_6{}^5}+\frac{\sin5\varphi'_6}{R'_6{}^5}\Bigr)+\frac{5!}{2}(1-R_1\cos\varphi_1)\Bigl(\frac{\sin6\varphi_6}{R_6{}^6}+\frac{\sin6\varphi'_6}{R'_6{}^6}\Bigr)\Bigr\}$$

....

$$\tau/K=(D_1+D_2+D_3+\cdots\cdots+D_6+\cdots\cdots)/Db \qquad (18)$$

$$D_1=\frac{b}{2}R_1\cos\varphi_1\Bigl(\frac{\cos\varphi_1}{R_1}-\frac{\cos\varphi'_1}{R'_1}\Bigr)$$

$$D_2=\frac{1-2C}{4}\log\frac{R'_2}{R_2}+\frac{1}{4}R_1\cos\varphi_1\Bigl(\frac{\cos\varphi_2}{R_2}-\frac{\cos\varphi'_2}{R'_2}\Bigr)-\frac{1}{2}(1-R_1\cos\varphi_1)\Bigl(\frac{\cos2\varphi_2}{R_2{}^2}-\frac{\cos2\varphi'_2}{R'_2{}^2}\Bigr)$$

$$D_3=-CR_1\cos\varphi_1\Bigl(\frac{\cos\varphi_3}{R_3}-\frac{\cos\varphi'_3}{R'_3}\Bigr)-2!R_1\cos\varphi_1\Bigl(\frac{\cos3\varphi_3}{R_3{}^3}-\frac{\cos3\varphi'_3}{R'_3{}^3}\Bigr)-\frac{1-2C}{4}\log\frac{R'_3}{R_3}$$

$$+\frac{1}{4}R_1\cos\varphi_1\left(\frac{\cos\varphi_3}{R_3}-\frac{\cos\varphi'_3}{R'_3}\right)+\frac{1}{2}(1-R_1\cos\varphi_1)\left(\frac{\cos2\varphi_3}{R_3{}^2}-\frac{\cos2\varphi'_3}{R'_3{}^2}\right)$$

$$D_4=\frac{b}{2}R_1\cos\varphi_1\left(\frac{\cos\varphi_4}{R_4}-\frac{\cos\varphi'_4}{R'_4}\right)-\frac{2C}{b}\left\{\frac{1-2C}{4}\log\frac{R'_4}{R_4}+\frac{1}{4}R_1\cos\varphi_1\left(\frac{\cos\varphi_4}{R_4}-\frac{\cos\varphi'_4}{R'_4}\right)\right.$$
$$\left.-\frac{1}{2}(1-R_1\cos\varphi_1)\left(\frac{\cos2\varphi_4}{R_4{}^2}-\frac{\cos2\varphi'_4}{R'_4{}^2}\right)\right\}-\frac{2}{b}\left\{\frac{1-2C}{4}\left(\frac{\cos2\varphi_4}{R_4{}^2}-\frac{\cos2\varphi'_4}{R'_4{}^2}\right)+\frac{2!}{4}R_1\cos\varphi_1\left(\frac{\cos3\varphi_4}{R_4{}^3}\right.\right.$$
$$\left.\left.-\frac{\cos3\varphi'_4}{R'_4{}^3}\right)-\frac{3!}{2}(1-R_1\cos\varphi_1)\left(\frac{\cos4\varphi_4}{R_4{}^4}-\frac{\cos4\varphi'_4}{R'_4{}^4}\right)\right\}$$

$$D_5=\frac{b}{2}\left(\frac{4C^2}{b^2}-1\right)R_1c\cos\varphi_1\left(\frac{\cos\varphi_5}{R_5}-\frac{\cos\varphi'_5}{R'_5}\right)+\frac{4C}{b}2!R_1\cos\varphi_1\left(\frac{\cos3\varphi_5}{R_5{}^3}-\frac{\cos3\varphi'_5}{R'_5{}^3}\right)+\frac{2}{b}4!R_1\cos\varphi_1$$
$$\times\left(\frac{\cos5\varphi_5}{R_5{}^5}-\frac{\cos5\varphi'_5}{R'_5{}^5}\right)-\frac{2C}{b}\left\{-\frac{1-2C}{4}\log\frac{R'_5}{R_5}+\frac{1}{4}R_1\cos\varphi_1\left(\frac{\cos\varphi_5}{R_5}-\frac{\cos\varphi'_5}{R'_5}\right)+\frac{1}{2}\left(1\right.\right.$$
$$\left.\left.-R_1\cos\varphi_1\right)\left(\frac{\cos2\varphi_5}{R_5{}^2}-\frac{\cos2\varphi'_5}{R'_5{}^2}\right)\right\}-\frac{2}{b}\left\{-\frac{1-2C}{4}\left(\frac{\cos2\varphi_5}{R_5{}^2}-\frac{\cos2\varphi'_5}{R'_5{}^2}\right)+\frac{2!}{4}R_1\cos\varphi_1\left(\frac{\cos3\varphi_5}{R_5{}^3}\right.\right.$$
$$\left.\left.-\frac{\cos3\varphi'_5}{R'_5{}^3}\right)+\frac{3!}{2}(1-R_1\cos\varphi_1)\left(\frac{\cos4\varphi_5}{R_5{}^4}-\frac{\cos4\varphi'_5}{R'_5{}^4}\right)\right\}$$

$$D_6=-CR_1\cos\varphi_1\left(\frac{\cos\varphi_6}{R_6}-\frac{\cos\varphi'_6}{R'_6}\right)-2!R_1\cos\varphi_1\left(\frac{\cos3\varphi_6}{R_6{}^3}-\frac{\cos3\varphi'_6}{R'_6{}^3}\right)+\left(\frac{4C^2}{b^2}-1\right)\left\{\frac{1-2C}{4}\log\frac{R'_6}{R_6}\right.$$
$$\left.+\frac{1}{4}R_1\cos\varphi_1\left(\frac{\cos\varphi_6}{R_6}-\frac{\cos\varphi'_6}{R'_6}\right)-\frac{1}{2}(1-R_1\cos\varphi_1)\left(\frac{\cos2\varphi_6}{R_6{}^2}-\frac{\cos2\varphi'_6}{R'_6{}^2}\right)\right\}$$
$$+\frac{8C}{b^2}\left\{\frac{1-2C}{4}\left(\frac{\cos2\varphi_6}{R_6{}^2}-\frac{\cos2\varphi'_6}{R'_6{}^2}\right)+\frac{2!}{4}R_1\cos\varphi_1\left(\frac{\cos3\varphi_6}{R_6{}^3}-\frac{\cos3\varphi'_6}{R'_6{}^3}\right)-\frac{3!}{2}(1-R_1\cos\varphi_1)\left(\frac{\cos4\varphi_6}{R_6{}^4}\right.\right.$$
$$\left.\left.-\frac{\cos4\varphi'_6}{R'_6{}^4}\right)\right\}+\left(\frac{2}{b}\right)^2\left\{3!\frac{1-2C}{4}\left(\frac{\cos4\varphi_6}{R_6{}^4}-\frac{\cos4\varphi'_6}{R'_6{}^4}\right)+\frac{4!}{4}R_1\cos\varphi_1\left(\frac{\cos5\varphi_6}{R_6{}^5}-\frac{\cos5\varphi'_6}{R'_6{}^5}\right)\right.$$
$$\left.-\frac{5!}{2}(1-R_1\cos\varphi_1)\left(\frac{\cos6\varphi_6}{R_6{}^6}-\frac{\cos6\varphi'_6}{R'_6{}^6}\right)\right\}$$

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3.4 多極座標による変位成分

$K'=P/\pi E$ とおけば，変位 u，v はつぎのように表わされる。

$$u/K'=(1+\nu)\left\{E_1+E_2+E_3+\cdots+E_6+\cdots-\nu(F_1+F_2+F_3+\cdots+F_6+\cdots)\right\}/Db \qquad (19)$$

$$E_1=\frac{b}{2}\left\{R_1(-\cos\varphi_1\log R_1+\sin\varphi_1\cdot\varphi_1+\cos\varphi_1)-R'_1(-\cos\varphi'_1\log R_1'+\sin\varphi'_1\cdot\varphi'_1+\cos\varphi'_1)\right.$$
$$\left.+R_1\cos\varphi_1\log\frac{R'_1}{R_1}\right\}$$

$$E_2=-\frac{C-1}{2}\left\{R_2(-\cos\varphi_2\log R_2+\sin\varphi_2\cdot\varphi_2+\cos\varphi_2)-R'_2(-\cos\varphi'_2\log R'_2\right.$$
$$\left.+\sin\varphi'_2\cdot\varphi'_2+\cos\varphi'_2)\right\}-\frac{1}{4}R_3\cos\varphi_3\log\frac{R'_2}{R_2}+\frac{1}{2}(1-R_1\cos\varphi_1)\left(\frac{\cos\varphi_2}{R_2}-\frac{\cos\varphi'_2}{R'_2}\right)$$

$$E_3=-C\left\{R_3(-\cos\varphi_3\cdot\log R_3+\sin\varphi_3\cdot\varphi_3+\cos\varphi_3)-R'_3(-\cos\varphi'_3\log R'_3+\sin\varphi'_3\cdot\varphi'_3+\cos\varphi'_3)\right.$$
$$\left.+R_1\cos\varphi_1\log\frac{R'_3}{R_3}\right\}+\left(\frac{\cos\varphi_3}{R_3}-\frac{\cos\varphi'_3}{R'_3}\right)-R_1\cos\varphi_1\left(\frac{\cos2\varphi_3}{R_3{}^2}-\frac{\cos2\varphi'_3}{R'_3{}^2}\right)-\frac{C-1}{2}\left\{R_3\left(\right.\right.$$
$$\left.\left.-\cos\varphi_3\log R_3+\sin\varphi_3\cdot\varphi_3+\cos\varphi_3\right)-R'_3(-\cos\varphi'_3\log R'_3+\sin\varphi'_3\cdot\varphi'_3+\cos\varphi'_3)\right\}$$
$$+\frac{1}{4}R_3\cos\varphi_3\log\frac{R'_3}{R_3}+\frac{1}{2}(1-R_1\cos\varphi_1)\left(\frac{\cos\varphi_3}{R_3}-\frac{\cos\varphi'_3}{R'_3}\right)$$

$$E_4=\frac{b}{2}\left\{R_4(-\cos\varphi_4\cdot\log R_4+\sin\varphi_4\cdot\varphi_4+\cos\varphi_4)-R'_4(-\cos\varphi'_4\cdot\log R'_4\right.$$
$$\left.+\sin\varphi'_4\cdot\varphi'_4+\cos\varphi'_4)-R_1\cos\varphi_1\log\frac{R'_4}{R_4}\right\}$$
$$+\frac{2C}{b}\left[\frac{C-1}{2}\left\{R_4(-\cos\varphi_4\cdot\log R_4+\sin\varphi_4\cdot\varphi_4+\cos\varphi_4)\right.\right.$$
$$\left.-R'_4(-\cos\varphi'_4\log R'_4+\sin\varphi'_4\cdot\varphi'_4+\cos\varphi'_4)\right\}+\frac{1}{4}R_3\cos\varphi_3\log\frac{R'_4}{R_4}-\frac{1}{2}(1-R_1\cos\varphi_1)\left(\frac{\cos\varphi_4}{R_4}\right.$$
$$\left.\left.-\frac{\cos\varphi'_4}{R'_4}\right)\right]-\frac{2}{b}\left\{\frac{C-1}{2}\left(\frac{\cos\varphi_4}{R_4}-\frac{\cos\varphi'_4}{R'_4}\right)-\frac{1}{4}R_3\cos\varphi_3\left(\frac{\cos2\varphi_4}{R_4{}^2}-\frac{\cos2\varphi'_4}{R'_4{}^2}\right)\right.$$

$$+\frac{2!}{2}(1-R_1\cos\varphi_1)\left(\frac{\cos3\varphi_4}{R_4^3}-\frac{\cos3\varphi'_4}{R'^3_4}\right)\Big\}$$

$$E_5=\frac{b}{2}\left(\frac{4C^2}{b^2}-1\right)\Big\{R_5(-\cos\varphi_5\log R_5+\sin\varphi_5\cdot\varphi_5+\cos\varphi_5)$$

$$-R'_5(-\cos\varphi'_5\log R'_5+\sin\varphi'_5\cdot\varphi'_5+\cos\varphi'_5)+R_1\cos\varphi_1\log\frac{R'_5}{R_5}\Big\}-\frac{4C}{b}\Big\{\left(\frac{\cos\varphi_5}{R_5}-\frac{\cos\varphi'_5}{R'_5}\right)$$

$$-R_1\cos\varphi_1\left(\frac{\cos2\varphi_5}{R_5^2}-\frac{\cos2\varphi'_5}{R'^2_5}\right)\Big\}-\frac{2}{b}\Big\{2!\left(\frac{\cos3\varphi_5}{R_5^3}-\frac{\cos3\varphi'_5}{R'^3_5}\right)-3!R_1\cos\varphi_1\left(\frac{\cos4\varphi_5}{R_5^4}-\frac{\cos4\varphi'_5}{R'^4_5}\right)\Big\}$$

$$+\frac{2C}{b}\Big[\frac{C-1}{2}\Big\{R_5(-\cos\varphi_5\log R_5+\sin\varphi_5\cdot\varphi_5+\cos\varphi_5)-R'_5(-\cos\varphi'_5\log R'_5+\sin\varphi'_5\cdot\varphi'_5$$

$$+\cos\varphi'_5)\Big\}-\frac{1}{4}R_3\cos\varphi_3\log\frac{R'_5}{R_5}-\frac{1}{2}(1-R_1\cos\varphi_1)\left(\frac{\cos\varphi_5}{R_5}-\frac{\cos\varphi'_5}{R'_5}\right)\Big]-\frac{2}{b}\Big\{\frac{C-1}{2}\Big(\frac{\cos\varphi_5}{R_5}$$

$$-\frac{\cos\varphi'_5}{R'_5}\Big)+\frac{1}{4}R_3\cos\varphi_3\left(\frac{\cos2\varphi_5}{R_5^2}-\frac{\cos2\varphi'_5}{R'^2_5}\right)+\frac{2!}{2}(1-R_1\cos\varphi_1)\left(\frac{\cos3\varphi_5}{R_5^3}-\frac{\cos3\varphi'_5}{R'^3_5}\right)\Big\}$$

$$E_6=-C\Big\{R_6(-\cos\varphi_6\log R_6+\sin\varphi_6\cdot\varphi_6+\cos\varphi_6)-R'_6(-\cos\varphi'_6\log R'_6+\mathrm{sih}\,\varphi'_6\cdot\varphi'_6$$

$$+\cos\varphi'_6)-R_1\cos\varphi_1\log\frac{R'_6}{R_6}\Big\}$$

$$+\left(\frac{\cos\varphi_6}{R_6}-\frac{\cos\varphi'_6}{R'_6}\right)+R_1\cos\varphi_1\left(\frac{\cos2\varphi_6}{R_6^2}-\frac{\cos2\varphi'_6}{R'^2_6}\right)-\left(\frac{4C^2}{b^2}-1\right)\Big[\frac{C-1}{2}\Big\{R_6(-\cos\varphi_6\log R_6$$

$$+\sin\varphi_6\cdot\varphi_6+\cos\varphi_6)-R'_6(-\cos\varphi'_6\log R'_6+\sin\varphi'_6\cdot\varphi'_6+\cos\varphi'_6)\Big\}+\frac{1}{4}R_3\cos\varphi_3\log\frac{R'_6}{R_6}$$

$$-\frac{1}{2}(1-R_1\cos\varphi_1)\left(\frac{\cos\varphi_6}{R_6}-\frac{\cos\varphi'_6}{R'_6}\right)\Big]+\frac{8C}{b^2}\Big\{\frac{C-1}{2}\left(\frac{\cos\varphi_6}{R_6}-\frac{\cos\varphi'_6}{R'_6}\right)-\frac{1}{4}R_3\cos\varphi_3\Big(\frac{\cos2\varphi_6}{R_6^2}$$

$$-\frac{\cos2\varphi'_6}{R'^2_6}\Big)+\frac{2!}{2}(1-R_1\cos\varphi_1)\left(\frac{\cos3\varphi_6}{R_6^3}-\frac{\cos3\varphi'_6}{R'^3_6}\right)\Big\}+\left(\frac{2}{b}\right)^2\Big\{2!\frac{C-1}{2}\left(\frac{\cos3\varphi_6}{R_6^3}-\frac{\cos3\varphi'_6}{R'^3_6}\right)$$

$$-\frac{3!}{4}R_3\cos\varphi_3\left(\frac{\cos4\varphi_6}{R_6^4}-\frac{\cos4\varphi'_6}{R'^4_6}\right)+\frac{4!}{2}(1-R_1\cos\varphi_1)\left(\frac{\cos5\varphi_6}{R_6^5}-\frac{\cos5\varphi'_6}{R'^5_6}\right)\Big\}$$

....

$$F_1=b\Big\{R_1(-\cos\varphi_1\log R_1+\sin\varphi_1\cdot\varphi_1+\cos\varphi_1)-R'_1(-\cos\varphi'_1\log R'_1+\sin\varphi'_1\cdot\varphi'_1+\cos\varphi'_1)\Big\}$$

$$F_2=\frac{1}{2}\Big\{R_2(-\cos\varphi_2\log R_2+\sin\varphi_2\cdot\varphi_2+\cos\varphi_2)-R'_2(-\cos\varphi'_2\log R'_2+\sin\varphi'_2\cdot\varphi'_2$$

$$+\cos\varphi'_2)\Big\}-\log\frac{R'_2}{R_2}$$

$$F_3=-\left(2C-\frac{1}{2}\right)\Big\{R_3(-\cos\varphi_3\log R_3+\sin\varphi_3\cdot\varphi_3+\cos\varphi_3)-R'_3(-\cos\varphi'_3\log R'_3$$

$$+\sin\varphi'_3\cdot\varphi'_3+\cos\varphi'_3)\Big\}+\log\frac{R'_3}{R_3}+2\left(\frac{\cos\varphi_3}{R_3}-\frac{\cos\varphi'_3}{R'_3}\right)$$

$$F_4=\left(b-\frac{C}{b}\right)\Big\{R_4(-\cos\varphi_4\log R_4+\sin\varphi_4\cdot\varphi_4+\cos\varphi_4)-R'_4(-\cos\varphi'_4\log R'_4+\sin\varphi'_4\cdot\varphi'_4$$

$$+\cos\varphi'_4)\Big\}+\frac{2C}{b}\log\frac{R'_4}{R_4}+\frac{2}{b}\Big\{\frac{1}{2}\left(\frac{\cos\varphi_4}{R_4}-\frac{\cos\varphi'_4}{R'_4}\right)+\left(\frac{\cos2\varphi_4}{R_4}-\frac{\cos2\varphi'_4}{R'_4}\right)\Big\}$$

$$F_5=\Big\{b\left(\frac{2C^2}{b^2}-1\right)-\frac{C}{b}\Big\}\Big\{R_5(-\cos\varphi_5\log R_5+\sin\varphi_5\cdot\varphi_5+\cos\varphi_5)-R'_5(-\cos\varphi'_5\log R'_5$$

$$+\sin\varphi'_5\cdot\varphi'_5+\cos\varphi'_5)\Big\}$$

$$-\frac{8C}{b}\left(\frac{\cos\varphi_5}{R_5}-\frac{\cos\varphi'_5}{R'_5}\right)-\frac{4\cdot2!}{b}\left(\frac{\cos3\varphi_5}{R_5^3}-\frac{\cos3\varphi'_5}{R'^3_5}\right)-\frac{2C}{b}\log\frac{R'_5}{R_5}$$

$$+\frac{2}{b}\Big\{\frac{1}{2}\left(\frac{\cos\varphi_5}{R_5}-\frac{\cos\varphi'_5}{R'_5}\right)-\left(\frac{\cos2\varphi_5}{R_5^2}-\frac{\cos2\varphi'_5}{R'^2_5}\right)\Big\}$$

$$F_6=\Big\{\frac{1}{2}\left(\frac{4C^2}{b^2}-1\right)-2C\Big\}\Big\{R_6(-\cos\varphi_6\log R_6+\sin\varphi_6\cdot\varphi_6+\cos\varphi_6)$$

$$-R'_6(-\cos\varphi'_6\log R'_6+\sin\varphi'_6\cdot\varphi'_6+\cos\varphi'_6)\Big\}+2\left(\frac{\cos\varphi_6}{R_6}-\frac{\cos\varphi'_6}{R'_6}\right)-\left(\frac{4C^2}{b^2}-1\right)\log\frac{R_6}{R'_6}$$

$$-\frac{8C}{b^2}\Big\{\frac{1}{2}\left(\frac{\cos\varphi_6}{R_6}-\frac{\cos\varphi'_6}{R'_6}\right)+\left(\frac{\cos2\varphi_6}{R_6^2}-\frac{\cos2\varphi'_6}{R'^2_6}\right)\Big\}-\left(\frac{2}{b}\right)^2\Big\{\frac{2!}{2}\left(\frac{\cos3\varphi_6}{R_6^3}-\frac{\cos3\varphi'_6}{R'^3_6}\right)$$

$$+3!\left(\frac{\cos4\varphi_6}{R_6^4}-\frac{\cos4\varphi'_6}{R'^4_6}\right)\Big\}$$

....

$$v/K' = -\ (1+\nu)\ \Big\{G_1+G_2+G_3+\cdots+G_6+\cdots+\nu\,(H_1+H_2+H_3+\cdots+H_6+\cdots)\Big\}/Db \qquad (20)$$

$$G_1 = \frac{b}{2}\Big[-2\Big\{R_1\ (\cos\varphi_1\cdot\varphi_1+\sin\varphi_1\log R_1-\sin\varphi_1)+R'_1\ (\cos\varphi'_1\cdot\varphi'_1$$
$$+\sin\varphi'_1\log R'_1-\sin\varphi'_1)\Big\}+R_1\cos\varphi_1\ (\varphi_1+\varphi'_1)\Big]$$

$$G_2 = \frac{1+2C}{4}\Big\{R_2\ (\cos\varphi_2\cdot\varphi_2+\sin\varphi_2\log R_2-\sin\varphi_2)+R'_2\ (\cos\varphi'_2\cdot\varphi'_2+\sin\varphi'_2\log R'_2-\sin\varphi'_2)\Big\}$$
$$-\frac{1}{4}R_4\cos\varphi_4\ (\varphi_2+\varphi'_2)-\frac{1}{2}\ (1-R_1\cos\varphi_1)\Big(\frac{\sin\varphi_2}{R_2}+\frac{\sin\varphi'_2}{R'_2}\Big)$$

$$G_3 = -C\Big[-2\Big\{R_3\ (\cos\varphi_3\cdot\varphi_3+\sin\varphi_3\log R_3-\sin\varphi_3)+R'_3\ (\cos\varphi'_3\cdot\varphi'_3+\sin\varphi'_3\log R'_3$$
$$-\sin\varphi'_3)\Big\}+R_1\cos\varphi_1\ (\varphi_3+\varphi'_3)\Big]-\Big\{2\Big(\frac{\sin\varphi_3}{R_3}+\frac{\sin\varphi'_3}{R'_3}\Big)+R_1\cos\varphi_1\Big(\frac{\sin2\varphi_3}{R_3^2}+\frac{\sin2\varphi'_3}{R'^2_3}\Big)\Big\}$$
$$-\frac{1+2C}{4}\Big\{R_3\ (\cos\varphi_3\cdot\varphi_3+\sin\varphi_3\log R_3-\sin\varphi_3)+R'_3\ (\cos\varphi'_3\ \varphi_3+\sin\varphi'_3\log R'_3-\sin\varphi'_3)\Big\}$$
$$-\frac{1}{4}R_4\cos\varphi_4\ (\varphi_3+\varphi'_3)+\frac{1}{2}\ (1-R_1\cos\varphi_1)\ \Big(\frac{\sin\varphi_3}{R_3}+\frac{\sin\varphi'_3}{R'_3}\Big)$$

$$G_4 = \frac{b}{2}\Big[2\Big\{R_4\ (\cos\varphi_4\cdot\varphi_4+\sin\varphi_4\log R_4-\sin\varphi_4)+R'_4\ (\cos\varphi'_4\cdot\varphi'_4+\sin\varphi'_4\log R'_4$$
$$-\sin\varphi'_4)\Big\}+R_1\cos\varphi_1\ (\varphi_4+\varphi'_4)\Big]-\frac{2C}{b}\Big[\frac{1+2C}{4}\Big\{R_4\ (\cos\varphi_4\cdot\varphi_4+\sin\varphi_4\log R_4-\sin\varphi_4)$$
$$+R'_4\ (\cos\varphi'_4\cdot\varphi'_4+\sin\varphi'_4\log R'_4-\sin\varphi'_4)\Big\}-\frac{1}{4}R_4\cos\varphi_4(\varphi_4+\varphi'_4)-\frac{1}{2}\ (1-R_1\cos\varphi_1)\Big(\frac{\mathrm{siu}\,\varphi_4}{R_4}$$
$$+\frac{\sin\varphi'_4}{R'_4}\Big)\Big]+\frac{2}{b}\Big\{\frac{1+2C}{4}\Big(\frac{\sin\varphi_4}{R_4}+\frac{\sin\varphi'_4}{R'_4}\Big)+\frac{1}{4}R_4\cos\varphi_4\Big(\frac{\sin2\varphi_4}{R_4^2}+\frac{\sin2\varphi'_4}{R'^2_4}\Big)$$
$$+\frac{2!}{2}\ (1-R_1\cos\varphi_1)\Big(\frac{\sin3\varphi_4}{R_4^3}+\frac{\sin3\varphi'_4}{R'^3_4}\Big)\Big\}$$

$$G_5 = \frac{b}{2}\Big(\frac{4C^2}{b^2}-1\Big)\Big[-2\Big\{R_5\ (\cos\varphi_5\cdot\varphi_5+\sin\varphi_5\log R_5-\sin\varphi_5)+R'_5\ (\cos\varphi'_5\cdot\varphi'_5$$
$$+\sin\varphi'_5\log R'_5-\sin\varphi'_5)\Big\}+R_1\cos\varphi_1\ (\varphi_5+\varphi'_5)\Big]+\frac{4C}{b}\Big\{2\Big(\frac{\sin\varphi_5}{R_5}+\frac{\sin\varphi'_5}{R'_5}\Big)$$
$$+R_1\cos\varphi_1\Big(\frac{\sin2\varphi_5}{R_5^2}+\frac{\sin2\varphi'_5}{R'^2_5}\Big)\Big\}+\frac{2}{b}\Big\{2\cdot2!\Big(\frac{\sin3\varphi_5}{R_5^3}+\frac{\sin3\varphi'_5}{R'^3_5}\Big)+3!\,R_1\cos\varphi_1\Big(\frac{\sin4\varphi_5}{R_5^4}$$
$$+\frac{\sin4\varphi'_5}{R'^4_5}\Big)\Big\}-\frac{2C}{b}\Big[-\frac{1+2C}{4}\Big\{R_5\ (\cos\varphi_5\cdot\varphi_5+\sin\varphi_5\log R_5-\sin\varphi_5)+R'_5\ (\cos\varphi'_5\cdot\varphi'_5$$
$$+\sin\varphi'_3\log R'_5-\sin\varphi'_5)\Big\}-\frac{1}{4}R_4\cos\varphi_4\ (\varphi_5+\varphi'_5)+\frac{1}{2}\ (1-R_1\cos\varphi_1)\ \Big(\frac{\sin\varphi_5}{R_5}+\frac{\sin\varphi'_5}{R'_5}\Big)\Big]$$
$$-\frac{2}{b}\Big\{\frac{1+2C}{4}\Big(\frac{\sin\varphi_5}{R_5}+\frac{\sin\varphi'_5}{R'_5}\Big)-\frac{1}{4}R_4\cos\varphi_4\Big(\frac{\sin2\varphi_5}{R_5^2}+\frac{\sin2\varphi'_5}{R'^2_5}\Big)$$
$$+\frac{2!}{2}\ (1-R_1\cos\varphi_1)\Big(\frac{\sin3\varphi_5}{R_5^3}+\frac{\sin3\varphi'_5}{R'^3_5}\Big)\Big\}$$

$$G_6 = -C\Big[2\Big\{R_6\ (\cos\varphi_6\cdot\varphi_6+\sin\varphi_6\log R_6-\sin\varphi_6)+R'_6\ (\cos\varphi'_6\cdot\varphi'_6+\sin\varphi'_6\log R'_6$$
$$-\sin\varphi'_6)\Big\}+R_1\cos\varphi_1\ (\varphi_6+\varphi'_6)\Big]+\Big\{2\Big(\frac{\sin\varphi_6}{R_6}+\frac{\sin\varphi'_6}{R'_6}\Big)-R_1\cos\varphi_1\Big(\frac{\sin2\varphi_6}{R_6^2}+\frac{\sin2\varphi'_6}{R'^2_6}\Big)\Big\}$$
$$-\Big(\frac{4C^2}{b^2}-1\Big)\Big[-\frac{1+2C}{4}\Big\{R_6\ (\cos\varphi_6\cdot\varphi_6+\sin\varphi_6\log R_6-\sin\varphi_6)+R'_6\ (\cos\varphi'_6\cdot\varphi'_6$$
$$+\sin\varphi'_6\cdot\log R'_6-\sin\varphi'_6)\Big\}+\frac{1}{4}R_4\cos\varphi_4\ (\varphi_6+\varphi'_6)+\frac{1}{2}\ (1-R_1\cos\varphi_1)\ \Big(\frac{\sin\varphi_6}{R_6}+\frac{\sin\varphi'_6}{R'_6}\Big)\Big]$$
$$-\frac{8C}{b^2}\Big\{\frac{1+2C}{4}\ \Big(\frac{\sin\varphi_6}{R_6}+\frac{\sin\varphi'_6}{R'_6}\Big)+\frac{1}{4}R_4\cos\varphi_4\Big(\frac{\sin2\varphi_6}{R_6^2}+\frac{\sin2\varphi'_6}{R'^2_6}\Big)+\frac{2!}{2}\ (1-R_1\cos\varphi_1)\Big(\frac{\sin3\varphi_6}{R_6^3}$$
$$+\frac{\sin3\varphi'_6}{R'^3_6}\Big)\Big\}-\Big(\frac{2}{b}\Big)^2\Big\{2!\,\frac{1+2C}{4}\Big(\frac{\sin3\varphi_6}{R_6^3}+\frac{\sin3\varphi'_6}{R'^3_6}\Big)+\frac{3!}{4}R_4\cos\varphi_4\Big(\frac{\sin4\varphi_6}{R_6^4}+\frac{\sin4\varphi'_6}{R'^4_6}\Big)$$
$$+\frac{4!}{2}\ (1-R_1\cos\varphi_1)\Big(\frac{\sin5\varphi_6}{R_6^5}+\frac{\sin5\varphi'_6}{R'^5_6}\Big)\Big\}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$H_1 = b\Big\{R_1\ (\cos\varphi_1\cdot\varphi_1+\sin\varphi_1\log R_1-\sin\varphi_1)+R'_1\ (\cos\varphi'_1\cdot\varphi'_1+\sin\varphi'_1\cdot\log R'_1-\sin\varphi'_1)\Big\}$$

$$H_2 = -\frac{1}{2}\Big\{R_2\ (\cos\varphi_2\cdot\varphi_2 + \sin\varphi_2\cdot\log R_2 - \sin\varphi_2) + R'_2\ (\cos\varphi'_2\cdot\varphi'_2 + \sin\varphi'_2\cdot\log R'_2 - \sin\varphi'_2)\Big\} + (\varphi_2 + \varphi'_2)$$

$$H_3 = \left(-2C+\frac{1}{2}\right)\Big\{R_3\ (\cos\varphi_3\cdot\varphi_3 + \sin\varphi_3 \log R_3 - \sin\varphi_3) + R'_3\ (\cos\varphi'_3\cdot\varphi'_3 + \sin\varphi'_3\cdot\log R'_3 - \sin\varphi'_3)\Big\} + (\varphi_3+\varphi'_3) + 2\left(\frac{\sin\varphi_3}{R_3} + \frac{\sin\varphi'_3}{R'^3_3}\right)$$

$$H_4 = -\left(b-\frac{C}{b}\right)\Big\{R_4\ (\cos\varphi_4\cdot\varphi_4 + \mathrm{sing}\,\varphi_4\cdot\log R_4 - \sin\varphi_4) + R'_4\ (\cos\varphi'_4\cdot\varphi'_4 + \sin\varphi'_4\cdot\log R'_4 - \sin\varphi'_4)\Big\} - \frac{2C}{b}(\varphi_4+\varphi'_4) - \frac{2}{b}\left\{\frac{1}{2}\left(\frac{\sin\varphi_4}{R_4} + \frac{\sin\varphi'_4}{R'_4}\right) + \left(\frac{\sin 2\varphi_4}{R_4^2} + \frac{\sin 2\varphi'_4}{R'^2_4}\right)\right\}$$

$$H_5 = \left\{b\left(\frac{4C^2}{b^2}-1\right) - \frac{C}{b}\right\}\Big\{R_5\ (\cos\varphi_5\cdot\varphi_5 + \sin\varphi_5\cdot\log R_5 - \sin\varphi_5) + R'_5\ (\cos\varphi'_5\cdot\varphi'_5 + \sin\varphi'_5\cdot\log R'_5 - \sin\varphi'_5)\Big\} - \frac{2C}{b}(\varphi_5+\varphi'_5) - \frac{8C}{b}\left(\frac{\sin\varphi_5}{R_5} + \frac{\sin\varphi'_5}{R'_5}\right) - \frac{4\cdot 2!}{b}\left(\frac{\sin 3\varphi_5}{R_5^3} + \frac{\sin 3\varphi'_5}{R'^3_5}\right) + \frac{2}{b}\left\{\frac{1}{2}\left(\frac{\sin\varphi_5}{R_5} + \frac{\sin\varphi'_5}{R'_5}\right) - \left(\frac{\sin 2\varphi_5}{R_5^2} + \frac{\sin 2\varphi'_5}{R'^2_5}\right)\right\}$$

$$H_6 = \left\{2C - \frac{1}{2}\left(\frac{4C^2}{b^2}-1\right)\right\}\Big\{R_6\ (\cos\varphi_6\cdot\varphi_6 + \sin\varphi_6\cdot\log R_6 - \sin\varphi_6) + R'_6\ (\cos\varphi'_6\cdot\varphi'_6 + \sin\varphi'_6\cdot\log R'_6 - \sin\varphi'_6)\Big\} + \left(\frac{4C^2}{b^2}-1\right)(\varphi_6+\varphi'_6) - 2\left(\frac{\sin\varphi_6}{R_6} + \frac{\sin\varphi'_6}{R'_6}\right) + \frac{8C}{b^2}\left\{\frac{1}{2}\left(\frac{\sin\varphi_6}{R_6} + \frac{\sin\varphi'_6}{R'_6}\right) + \left(\frac{\sin 2\varphi_6}{R_6^2} + \frac{\sin 2\varphi'_6}{R'^2_6}\right)\right\} + \left(\frac{2}{b}\right)^2\left\{\frac{2!}{2}\left(\frac{\sin 3\varphi_6}{R_6^3} + \frac{\sin 3\varphi'_6}{R'^3_6}\right) + 3!\left(\frac{\sin 4\varphi_6}{R_6^4} + \frac{\sin 4\varphi'_6}{R'^4_6}\right)\right\}$$

………………………………

4. 応力成分，変位成分の数値計算

σ_y/K，τ/K，u/K'，v/K' は各式(17)(18)(19)(20)において，項数の増加と共に，境界条件をよく満足するようになるが，収束はおそくなる．これは，マクローリン展開が，局所近似式である事に由来し，$\lambda=0$ 近傍ではよい近似値を与えるが，λ が大になる場合に誤差も大になるためである．そこで本論文では，以下に述べる方法を使用する．

式(17)の σ_y/K において，負荷端Y=1.0では，$-2C_1/b$ は，境界条件を表わし，また $C_2=-C_3$，$C_4=-C_5$，……となる．式(18)の τ/K も同様である．一方Y=0.0すなわち固定端において，式(19)の u/K' では，$(E_1-\nu F_1) = -(E_2-\nu F_2)$，…，$(E_5-\nu F_6) = -(E_6-\nu F_6)$，…となる．式(20)の v/K' においても同様である．すなわち，奇数項まで取れば負荷端，$Y=1.0$ の境界条件は完全に満足され，偶数項まで取れば固定端 $Y=0.0$ の境界条件は完全に満足される．しかるに，3節で述べたように，式(14)の応力関数の各項 A_1，A_2，…はそれぞれ単独に式(1)を満足するから，これらの線形結合もまた式(1)の解である．そこで式(16,(17)(18)(19)(20)を次のように有限項で表わすことにする．k_1，k_2を任意定数とすれば，

$$\sigma_x/K = \Big\{(B_1+B_2) + k_1(B_3+B_4) + k_2(B_5+B_6)\Big\}/Db \qquad (21)$$

$$\sigma_y/K = -\Big\{(C_1+C_2) + k_1(C_3+C_4) + k_2(C_5+C_6)\Big\}/Db \qquad (22)$$

$$\tau/K = \Big\{(D_1+D_2) + k_1(D_3+D_4) + k_2(D_5+D_6)\Big\}/Db \qquad (23)$$

$$u/K' = (1+\nu)\Big[(E_1-\nu F_1) + (E_2-\nu F_2) + k_1\Big\{(E_3-\nu F_3) + (E_4-\nu F_4)\Big\} + k_2\Big\{(E_5-\nu F_5) + (E_6-\nu F_6)\Big\}\Big]/Db \qquad (24)$$

$$v/K' = -(1+\nu)\Big[(G_1+\nu H_1) + (G_2+\nu H_2) + k_1\Big\{(G_3+\nu H_3) + (G_4+\nu H_4)\Big\} + k_2\Big\{(G_5+\nu H_5) + (G_6+\nu H_6)\Big\}\Big]/Db \qquad (25)$$

いま，ポアソン比 $\nu=0.3$ とし，計算範囲を $X\leqq 1.0$ とし，$Y=1.0$，$X=1.0$ で境界条件を完全に満足するように k_1，k_2を定めると，$k_1=0.818060$，$k_2=0.301072$ となる．この場合負荷端$Y=1.0$ における σ_y/K，τ/K は Table 1のようになる．また $Y=0.5$における本論文の近似値と数値積分値とを比較すると Table 2 を得る．数値積分は，シンプソンの公式で（$\Delta\lambda=0.1$）求めた．これより近似計算値が実用上十分な精度を有する事が判る．計算所要時間は，本論文の場合，X，Y に無関係で3応力，2変位成分値を求めるのに約0.4秒かかり，数値積分（小数点以下4桁まで）で3応力成分を求めた場合，例えば $X=0.2$の場合，$Y=0.0$ で約2秒，$Y=0.5$ で約3.5秒であるが，$Y=1.0$ では40分でも収束はしていない．(HITAC 8350)

Table. 1. Stress on the boundary, Y=1.0

	X	0.0	0.2	0.4	0.6	0.8	1.0
σ_y/K	Boundary condition	−3.1416	−3.1416	−3.1416	0.0000	0.0000	0.0000
	Approximate values	−3.1629	−3.1613	−3.1570	−0.0096	−0.0040	0.0000
τ/K	Boundary condition	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	Approximate values	0.0000	−0.0032	−0.0052	−0.0053	−0.0030	0.0000

Table. 2. Errors of stress on Y=0.5

	X	0.0	0.2	0.4	0.6	0.8	1.0
σ_x/K	Approximate values	−0.2667	−0.2772	−0.3569	−0.4915	−0.5155	−0.4254
	Numerical integration	−0.2672	−0.2778	−0.3575	−0.4923	−0.5165	−0.4265
	Errors	0.0005	0.0006	0.0006	0.0008	0.0010	0.0011
	Errors (%)	0.19	0.22	0.17	0.16	0.19	0.26
σ_y/K	Approximate values	−2.7737	−2.5870	−2.0040	−1.1854	−0.5447	−0.2054
	Numerical integration	−2.7543	−2.5692	−1.9904	−1.1774	−0.5422	−0.2071
	Errors	−0.0194	−0.0178	−0.0136	−0.0080	−0.0025	0.0017
	Errors (%)	0.70	0.69	0.68	0.68	0.46	0.82
τ/K	Approximate values	0.0000	0.2907	0.5346	0.5516	0.3591	0.1745
	Numerical integration	0.0000	0.2885	0.5306	0.5467	0.3540	0.1698
	Errors	0.0000	0.0022	0.0040	0.0049	0.0051	0.0047
	Errors (%)	0.00	0.76	0.75	0.90	1.44	2.77

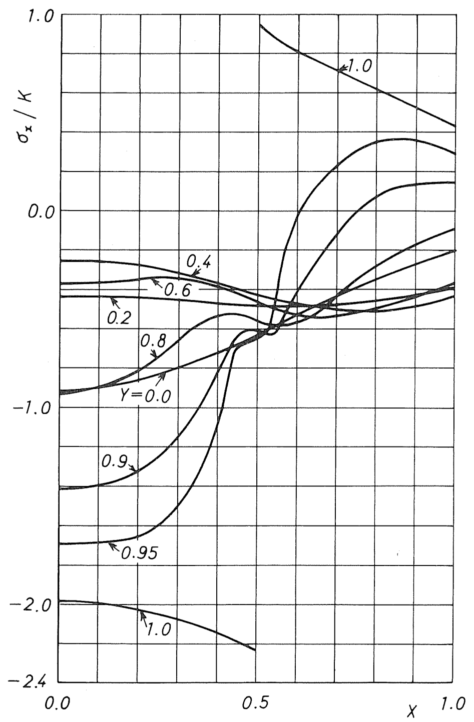

Fig. 4. Distribution of σ_x/K

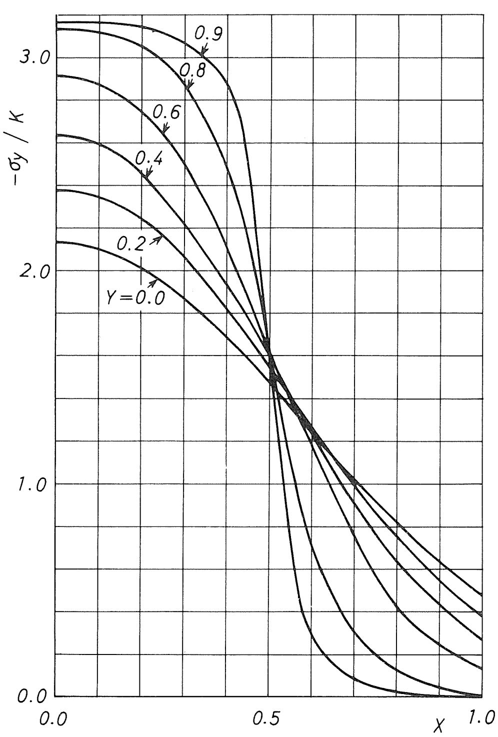

Fig. 5. Distribution of $-\sigma_y$/K

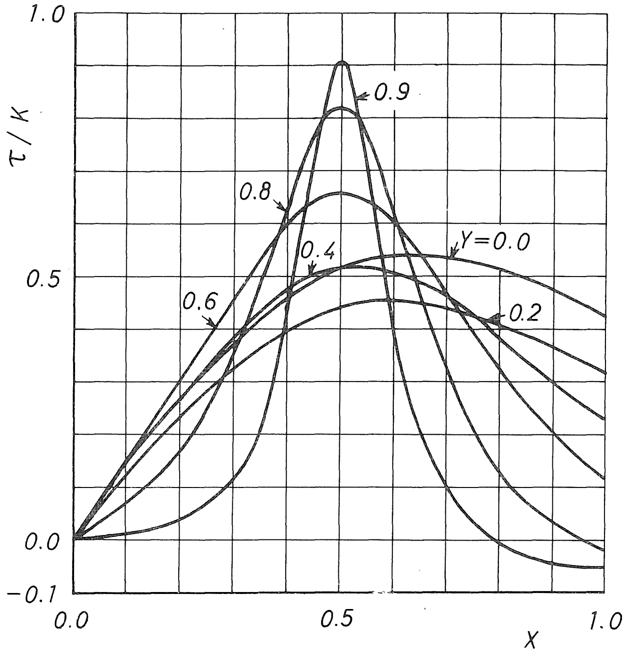

Fig. 6.　Distribution of τ/K

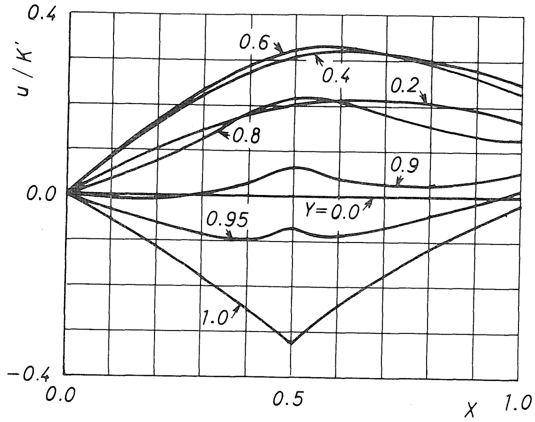

Fig. 7.　Distribution of u/K'

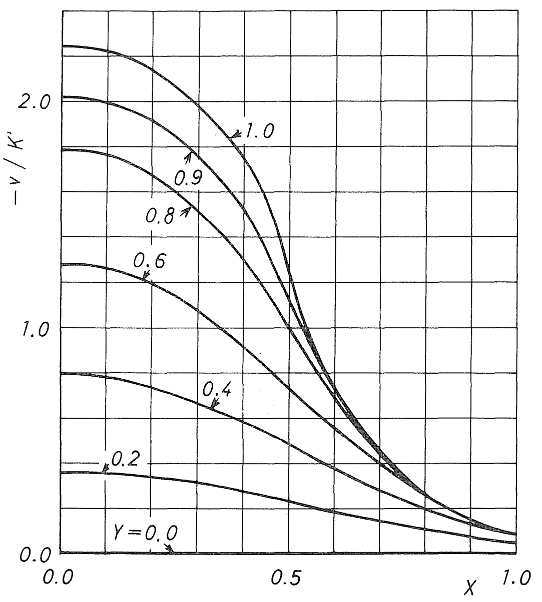

Fig. 8.　Distribution of −v/K'

5. 結　　言

弾性板の一端が，剛体床上に固定され，他端に，等分布荷重を受ける場合の応力解析を先に，座標軸に対称な荷重を受ける弾性帯板の場合に使用した解法と類似の方法を用いて行なった．無限積分形で表わされた応力関数の双曲線関数部分を，指数関数で表わし，分母をマクローリン展開し，これより各応力および変位成分を求めた．

マクローリン展開式は，局所近似式であり，本問題のように変数 λ の範囲が0～∞の場合には，必ずしも適当な方法ではないが，項別積分の各項が，それぞれ応力関数となり，したがってこれらの線形結合もまた応力関数であることを利用し，境界条件を近似的に満足するように，二元連立一次方程式を解く事により，実用上十分な精度の近似値を得た．数値積分法では容易に求め得ない負荷端での応力成分を本論文では短時間で求め得る事を示した．

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